Romanian Reports in Physics **75**, 704 (2023)

EVALUATING THE PROBABLE MAXIMUM PRECIPITATION. CASE STUDY FROM THE DOBROGEA REGION, ROMANIA

A. BĂRBULESCU*, C. MAFTEI

Transilvania University of Brasov, 5, Turnului Str., 900152, Brasov, Romania
E-mails: *alina.barbulescu@unitbv.ro*; *cemaftei@gmail.com*
* Author for correspondence: A. Barbulescu, E-mail: *alina.barbulescu@unitbv.ro*

Received January 3, 2023

Abstract. In this article, the Probable Maximum Precipitation (PMP) evaluation is done using a spatio-temporal approach – Diaconu's method – for a set of data series collected in the Dobrudja region for 41 years. The results are compared with those obtained by applying the classical method of Hershfield and the advantages of each method are emphasized.

Key words: PMP, Diaconu's Method, Hershfield, distribution.
DOI: https://doi.org/10.59277/RomRepPhys.2023.75.704

1. INTRODUCTION

According to the World Meteorological Organization 2009 – WMO 1045 – Manual on Estimation of Probable Maximum Precipitation (PMP) [1], PMP represents "the theoretical maximum precipitation for a given duration." This concept relies on the hypothesis that the estimated PMP value is so high that the probability that an event producing a rainfall amount higher than PMP is extremely small, so there is no risk associated with the exceedance of the PMP value.

The PMP determination is important for the Probable Maximum Flow (PMF) computation in the context of flood/flash flood estimation and dam or other's hydraulic structure design [2]. In the climate context, estimated PMPs can be the input of hydrological models that investigate possible values of PMF [3].

Many researchers [4–6] provide different methods and classifications for PMP evaluation. In the latest version of the WMO 1045 Manual [1], six methods for PMP estimation for medium or low latitude zones and two methods for large basin areas are presented. In an attempt to group them, six categories have been determined, based on the methodological background, among which are statistical, empirical, computational, and generalized [5, 6]. Another classification of these methods distinguishes only two classes – meteorological and statistical [4], the last relying on frequency analysis. It includes the method proposed by Hershfield in 1961 [7] and modified in 1965 [8], which proved to be successfully used for catchments under 1000 km^2. WMO [9] recommended utilizing the Hershfield method (denoted in the following PMP_H) only when long daily rainfall records are available at the meteorological stations.

For many years, the Hershfield method was the most common and simple approach for frequency precipitation analysis in the U.S. In 1999, Koutsoyiannis [10] used Generalized Extreme Value (GEV) distribution to fit the frequency factors calculated utilizing 2645 records initially utilized by Hershfield. Fitting a linear function of mean annual maximum rainfall and using it as a shape parameter in GEV, Koutsoyiannis retrieved the same result as by PMP_H.

The Gumbel distribution has been the most recommended model to analyze extreme precipitations. However, in 2004, Koutsoyiannis [11] proved that EV2 is a more consistent alternative to the Gumbel distribution. The Weibull distribution has also been suggested for modeling daily rainfall amounts [12, 13]. Recently, several authors have used more powerful methods (for example, the L-moment one). In this context, Bonnin [14] fit a GEV distribution to the precipitations in the USA. Pilon [15] selected the GEV for the data in Ontario, Canada. Deidda and Puliga [16] find some weak evidence for the Generalized Pareto distribution for left-censored records in Sardinia. Based on research results conducted in the Seyhan river basin in Turkey, Gumbel, log-Logistic, Pearson type III, log-Pearson type III, and log-normal-3 distributions were applied to the series of annual instantaneous flood peaks and annual peak daily precipitation.

In Romania, a spatial-temporal method was developed by Diaconu [17] and used to evaluate the PMP (denoted in the following PMP_D) for an area greater than 10000 km^2 utilizing a data set of 24 h maximum precipitation recorded at 19 stations in South Eastern part of the country. Still, only a few studies benefit from applying this methodology [17–20]. Therefore, in this article, we present the PMP evaluation by PMP_D and discuss the results *versus* those obtained by PMP_H on data series collected in the Dobrogea region, Romania, for 41 years. The advantages of each method are also emphasized. This approach is new, given that no article offers such a comparison.

2. DATA SERIES AND METHODOLOGY

2.1. DATA SERIES

The Dobrogea region (Fig. 1), with a surface of 15700 km^2, is situated in the southeastern part of Romania, between the Black Sea, the Danube River, and the Danube Delta. From a climate viewpoint, the region is divided into two parts – a belt of about 70 km from the Littoral, influenced by the Sea, and a continental-temperate part in the rest. Last 50 years, the average annual temperature in the region has been about 11ºC. Dobrogea is the arider zone in Romania, with drought periods from four to six months. In the last twenty years, long periods without precipitations were followed by days with very high quantities of water in a single day. Extensive studies on the temperature and precipitation trend are presented in [21–25]. The studied series contains the maximum 24 hour precipitation recorded in the Dobrogea region, Romania. The number of years and the highest 24 h precipitation value recorded are given in Table 1.

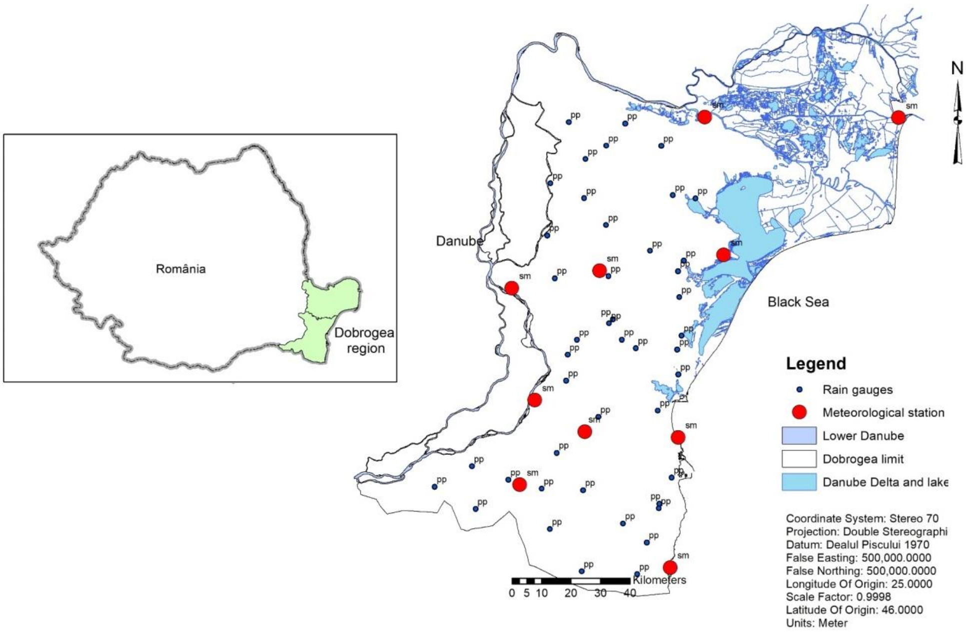

Fig. 1 – The map of the Dobrogea region.

Table 1

The locations, series length (years), and the highest 24 h rainfall (mm)

ID	Location	Latitude (degree)	Longitude (degree)	Elevation (m)	no of years	Highest observed 24h rainfall (mm)
1	Adamclisi	44.08	28.00	158	40	83.3
2	Cernavoda	44.21	28.03	87.17	41	73.6
3	Medgidia	44.15	28.16	69.54	36	84.6
4	Harsova	44.41	27.57	37.51	39	96.7
5	Corugea	44.44	28.20	219.2	36	114.8
6	Tulcea	45.11	28.49	4.36	36	134.5
7	Constanta	44.13	28.38	12.8	41	201
8	Mangalia	43.49	28.35	6	40	146
9	Jurilovca	44.46	28.53	37.65	21	79.9
10	Sulina	45.09	29.39	2.08	40	84.9

2.2. THE DIACONU'S METHOD

Diaconu's method is presented here based on [17–19, 26]. This is a spatio-temporal analysis of the 24 hour design rainfall.

To have an image of the studied data, the basic statistics of the series (the mean, coefficient of variation, and skewness) recorded at each site were first computed. Then, empirical probabilities and the most appropriate distribution function have been fitted to the data and tested using the Anderson-Darling [27] and chi-square goodness of fit tests [28].

After fitting an empirical distribution (Weibull, for example), the values at the probabilities of 2%, 5%, 10%, 20%, 50%, 80%, 90%, and 95% are computed by linear interpolation. A series of values at a specific temporal probability, p%T, is denoted by $X_{p\%T}$. For example, $X_{20\%T}$ is formed by 10 values, each one corresponding to a location, computed for the 20% temporal probability.

Denote by p%S the value of maximum precipitation with the property that the exceedance probability of p% can be exceeded on p% of the studied area. Thus, p%S has a spatial significance. Therefore, one may speak about maximum daily precipitation values with a temporal probability p%T and a spatial probability p%S. These values will be denoted in the following by $X_{p\%T \cap p\%S}$.

In Fig. 2, the value for a temporal probability of 10% (76 mm) has a spatial probability of 20%. Also, the maximum precipitation with a temporal exceedance probability of 10% and the spatial probability of 50% is 67 mm. Any other $X_{p\%T}$ series could be analyzed similarly.

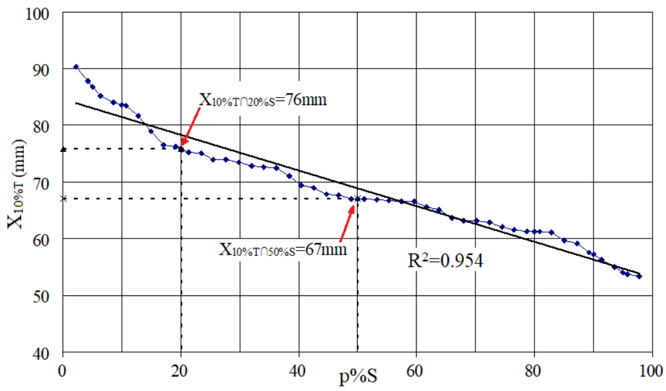

Fig. 2 – The chart of the $X_{10\%T}$ series.

The values for 2%, 5%, 10%, 20%, 50%, 80%, 90%, 95% spatial probabilities were extracted (by linear interpolation) for each series $X_{p\%T}$, and utilized for building the spatio – temporal probabilities' charts. There are two options to draw the diagrams, depending on the values on the Ox axis: spatial probability, p%S, or temporal p%T (Figs. 3 and 4).

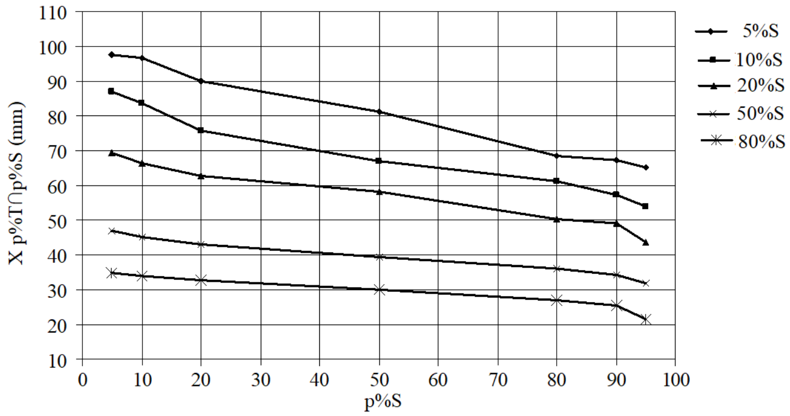

Fig. 3 – Representation of couples (p%S, $\boldsymbol{X}_{p\%T\cap p\%S}$).

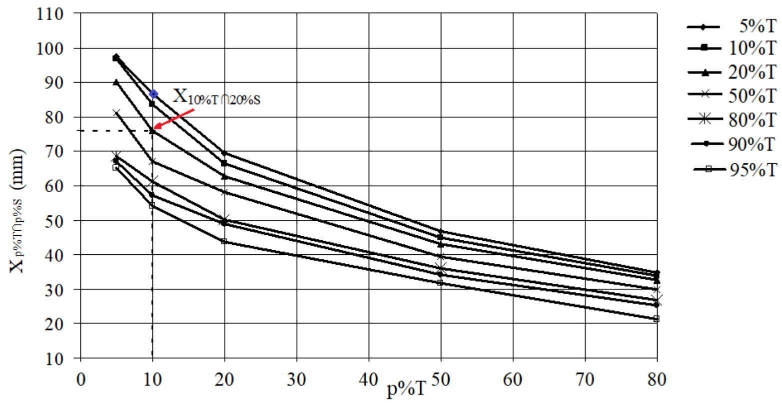

Fig. 4 – Representation of the couples (p%T, $X_{p\%T\cap p\%S}$).

Extrapolation of rare events (with probabilities of 0.01%, 0.05%, 0.1%) must be done using some theoretical distribution functions (for example, Pareto). An extrapolation example is presented in Fig. 5. Supposing that the surface of the studied region is F_z =10000 km^2, denote by F_B a p%S of this surface:

$$F_B = \text{p\%S} \cdot F_z \,. \tag{1}$$

F_B represents the surface of the region where the precipitation is higher than $X_{p\%T}$. For example, the value $X_{10\%\cap 5\%} \approx 88$ mm (the blue dot in Fig. 4) would be representative of a surface $F_B = 5\% \cdot 10000 = 500\,\text{km}^2$. If the surface F_z is different, F_B varies accordingly, but the curve $X_{p\%T}$ could remain the same. Therefore, standardization is necessary. Values of 10, 100, and 1000 are considered for F_B and 5000, 10000, 40000, 80000 km^2 for F_z. The p%S are displayed in Table 2.

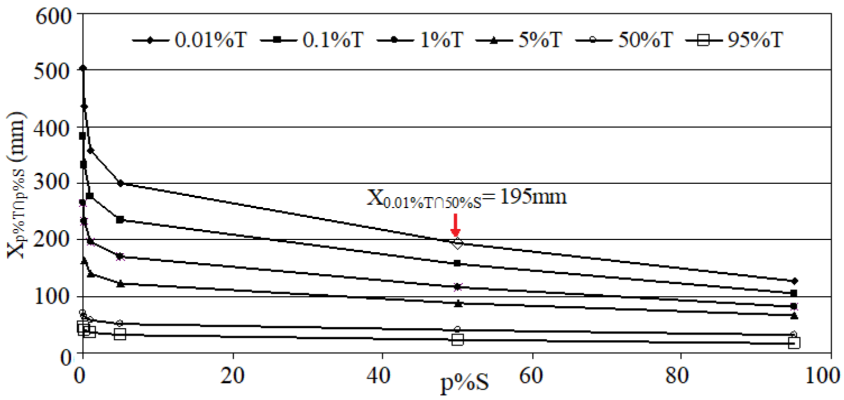

Fig. 5 – Extrapolation example.

Table 2

Locations, series length (years), and the highest 24 h rainfall (mm)

F_z (km[2]) / F_B (km[2])	5000	10000	40000	80000
10	0.2	0.1	0.025	0.0125
100	2	1	0.25	0.125
1000	20	10	2.5	1.25

The curves presented in Fig. 5 characterize the studied region, but they could also characterize extended regions. For this aim, the data presented in Fig. 5 have been used to extract $X_{0.01\%T}$ and $X_{5\%T}$ (Fig. 6) corresponding to the spatial probabilities. Their values are given in Table 3.

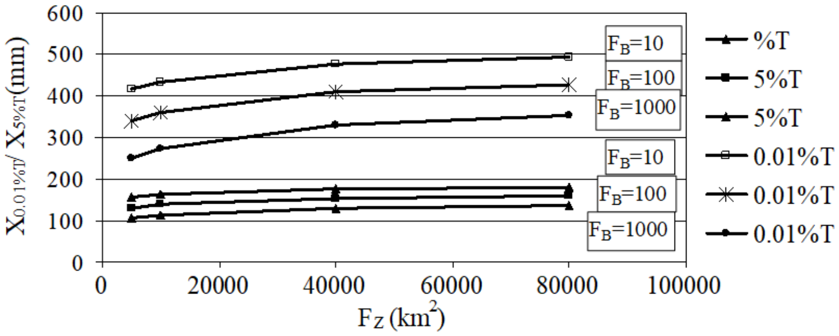

Fig. 6 – $X_{0.01\%T}$ and $X_{5\%T}$ corresponding to the spatial probabilities.

Remark that the maximum daily precipitation corresponding to the exceedance probability of 0.01% (the return period of 10000 years), on a surface of 10 km^2, tends to a limit of 500 mm and $h_{24h5\%}$ (the return period of 20 years) on a surface of 100 km^2 tends to a value of 170 mm.

Table 3

The 24 h maximal annual rainfall for p%T = 0.01% and 5% function of F_z and F_B

p%S	F_B (km^2)	F_z (km^2)	$X_{5\%}$ (mm)	$X_{0.01\%}$ (mm)
0.0125	10	80000	181	492
0.025	10	40000	178	476
0.1	10	10000	163	435
0.125	100	80000	161	427
0.2	10	5000	157	418
0.25	100	40000	155	410
1	100	10000	140	359
1.25	1000	80000	138	354
2	100	5000	131	340
2.5	1000	40000	129	330
10	1100	11000	112	274
20	1000	5000	106	249

2.3. THE HERSHFIELD METHOD

To estimate PMP_H using a set of maximum precipitation series (with *n* values each) based on the Hershfield method [7, 8], the following relations are utilized:

$$h_{max} = \overline{h_n} + k_{max} \cdot s_n , \quad (2)$$

where:

- h_{max} = PMP at a certain location when the duration is specified,
- $\overline{h_n}$ (s_n) = the average (standard deviation) value of the maximum annual precipitation series recorded at that location,
- k_m is the factor of frequency related to the chosen station, computed by:

$$k_m = (h_1 - \overline{h}_{n-1}) / s_{n-1} , \quad (3)$$

with:

- h_1 = the highest value of the series,
- $\overline{h}_{n-1}$ (s_{n-1}) has the same significance as $\overline{h_n}$ (s_n) for the series obtained after eliminating h_1 ,
- k_{max} = the maximum of k_m-s for all the observation sites in a zone.

Since Hershfield [8] found that k_m computed for over 2 600 series varied between 3 and 14.5, he adopted k_{max} = 15 for the PMP evaluation by this method.

3. RESULTS AND DISCUSSION

3.1. BASIC STATISTICS

The results of the statistical analysis of the 24 hours maximum precipitation (h_{24h}) series recorded at the ten meteorological stations are presented in Table 4, where

$$\Delta h_{24h} = h_{24h\,max} - h_{24h\,min}, \tag{4}$$

$$\Delta E = \frac{\Delta h_{24h}}{\overline{h_{24h}}}, \tag{5}$$

$$\varepsilon_0 = \left| \frac{2 \cdot \Delta h_{24h}}{h_{24h\,max} + h_{24h\,min}} \right|, \tag{6}$$

and $\overline{h_{24h}}$ is the series average.

Table 4

Descriptive statistics for the main meteorological stations from Dobrogea

	Adamclisi	Cernavoda	Constanta	Corugea	Harsova	Jurilovca	Mangalia	Medgidia	Sulina	Tulcea
Min	15	16.2	18.5	19.4	15.7	18.9	19.2	13.9	11.4	18
Mean	43.03	35.49	45.15	43.95	37.65	37.84	48.30	40.95	30.19	43.50
Max	83.3	73.6	201	114.8	96.7	79.9	146	84.6	84.9	134.5
Median	39.9	28.5	41.75	39.5	32.6	31.8	40.8	41.15	26.6	39.45
Std. dev.	15.87	16.22	29.54	22.77	18.66	16.56	26.59	15.01	16.58	22.84
Var.coef.(CV)	0.37	0.46	0.65	0.52	0.50	0.44	0.55	0.37	0.55	0.52
Skew. (Cs)	0.52	0.96	3.92	1.45	1.37	1.07	1.96	0.62	1.86	2.13
Cs/Cv	1.4	2.1	6.0	2.8	2.8	2.4	3.6	1.7	3.4	4.1
Δh_{24h}	68.3	57.4	182.5	95.4	81	61	126.8	70.7	73.5	116.5
ΔE	1.59	1.62	4.04	2.17	2.15	1.61	2.63	1.73	2.43	2.68
ε_0	1.39	1.28	1.66	1.42	1.44	1.23	1.54	1.44	1.53	1.53

The minimum varies between 11.4 and 19.4, with an average between 30.19 and 45.15, and standard deviations from 15.08 to 29.54. All series are right-skewed ($C_v > 0$). The coefficient of variation is between 0.37 and 0.65, with an average of 0.49, Δh_{24h} is inside the interval [61, 182.5] and ε_0 from 1.23 to 1.66. It results that the series set is homogeneous. For more details, the reader may see [22].

3.2. COMPUTATION OF PMP_D

To apply the PMP_D method, the Weibull empirical distribution has been fitted for all ten series. Hazen and Chegodaev models have also been fitted for comparison. Figure 7 indicates that the most appropriate empirical distribution for the Corugea series is Weibull ($R^2 = 0.9843$) in concordance with [19]. Other distributions have been fitted to the available series to find the best model utilized in the following. The results of the goodness-of-fit test (chi-square) at different significance levels (1%, 5%, and 10%) are given in Table 5.

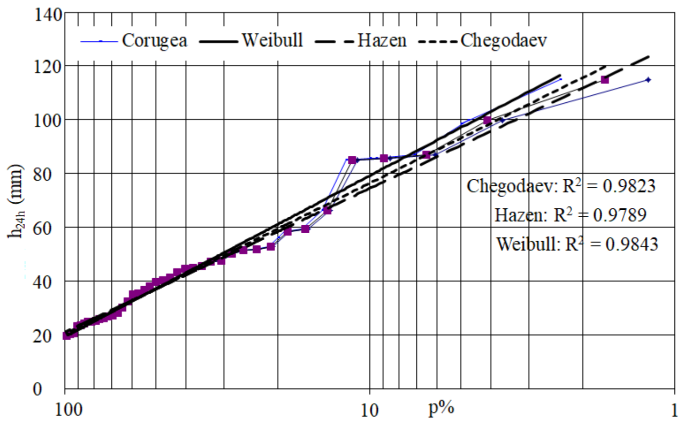

Fig. 7 – The empirical distribution for the Corugea series.

Table 5

Results of the chi-square test

Distribution \ Series	Adamclisi	Cernavoda	Constanta	Corugea	Harsova	Jurilovca	Mangalia	Medgidia	Sulina	Tulcea
EV2 - Max	A	A	A	R	A*	A**	A**	A	A	A**
GEV-Max	A	–	A	A	A*	R	A	A*	A	A
Gumbel	A	A	R	A	A**	R	A*	A	A	A*
Log Pearson III	A**	A	A	A	A	A**	A	R	A	A
Pearson III	A	–	R	A	A*	R	A	A*	A*	A
Weibull	A	A	R	A**	R	R	A	A*	A	A
Pareto	A**	A	A	A	A	A	A	A	A	A
EV2 Max (L-mom)	A	A	A	A	A	A**	A*	A	A	A
Gumbel (L-moment)	A	A	A	A	A	R	A	A	A	A
Pareto (L-mom)	A**	A	A	A	A	A	A	A**	A	A
Weibull (L-mom)	A	A	A	A	A	R	A**	A	A	A

Legend: R – reject, A – accept, A* – accept at the significance level of 1% and 5% and not statistically significant at 10%, A** – accept only at the significance level of 1%.

Remark that for the Jurilovca series, most models have been rejected. For Constanta, the Gumbel, Pearson III, and Weibull distributions are inappropriate. To keep a certain uniformity of the analysis, only the distributions accepted for all the data series (Pareto, EV2 Max (L-moment)), together with the LogPearson III and Weibull (L-moment) distributions have been used in the following steps (the last two have been chosen based on the goodness of fit). The 95% confidence intervals have been computed as well. Two examples are presented in Figs. 8 and 9.

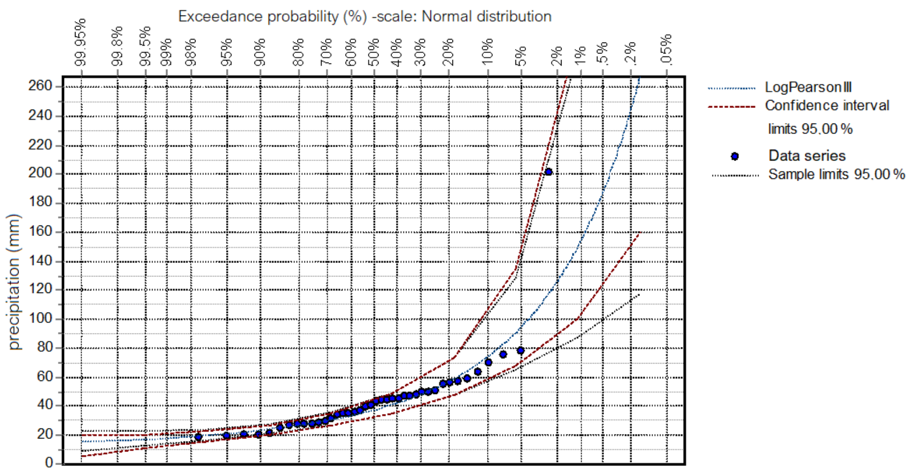

Fig. 8 – LogPearson III model for the Constanta series.

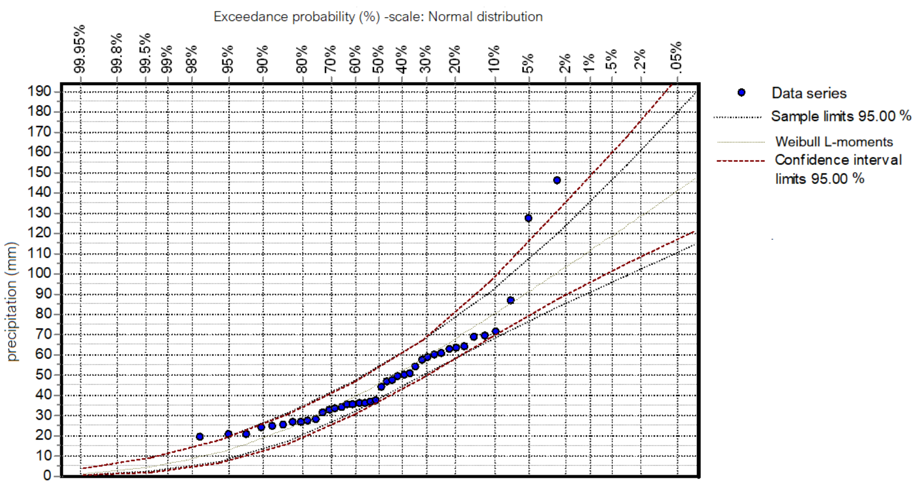

Fig. 9 – Weibull L-Moment model for the Mangalia series.

For the selected distribution function, the values corresponding to the 0.001%, 0.01%, 0.1%, 1%, 2%, 3%, 5%, 10%, 20%, 50%, 80%, and 90% exceedance probabilities have been computed (Tables 6–9). The four distribution functions describe very well the observed data.

Table 6

The maximum (mm) recorded and the 24 hour-rainfall values corresponding to different exceedance probabilities, computed based on the Pareto distribution function

Station	Max	Probability of exceedance (%)											
		0.001	0.01	0.10	1	2	3	5	10	20	50	80	90
Adamclisi	83.3	85.6	85.3	84.1	80.1	77.6	75.7	72.5	66.6	58.0	40.5	27.6	23.8
Cernavoda	73.6	103.3	100.1	94.0	82.1	76.7	73.1	67.8	59.3	48.9	31.6	20.8	17.9
Constanta	201.0	740.7	467.7	284.0	160.4	131.8	116.6	98.9	77.2	58.0	35.8	25.7	23.3
Corugea	114.8	197.1	177.7	152.4	152.4	152.4	152.4	90.1	75.7	60.1	37.5	37.5	21.6
Harsova	96.7	154.2	141.1	123.1	98.2	98.2	83.2	75.4	75.4	51.2	32.4	32.4	19.1
Jurilovca	79.9	114.8	110.0	101.7	87.2	81.0	76.9	71.0	61. 9	51.1	33.7	23.1	20.3
Mangalia	146.0	319.9	262.1	203.4	143.7	125.6	114.9	101.5	83.1	64.7	40.2	27.5	24.4
Medgidia	84.6	85.4	84.2	83.1	78.0	75.1	72.9	69.5	63.3	54.7	38.2	26.6	23.2
Sulina	84.9	187.7	156.6	123.7	89.0	78.1	71.7	63.5	52.7	40.7	25.2	17.1	15.0
Tulcea	134.5	305.5	243.1	183.7	126.9	110.4	100.8	88.8	72.8	57.0	36.4	26.0	26.0

Table 7

Values (mm) at different exceedance probabilities, computed based on the EV2-Max L-Moment distribution function

Station	Probability of exceedance (%)											
	0.001	0.01	0.10	1	2	3	5	10	20	50	80	90
Adamclisi	795.6	424.0	226.0	120.3	99.4	88.8	77.0	63.3	51.6	37.8	30.0	27.2
Cernavoda	1160.9	544.5	255.4	119.6	95.1	83.1	70.0	55.3	43.2	29.7	22.6	20.0
Constanta	1827.1	816.8	365.1	162.9	127.6	110.6	92.2	71.7	55.1	37.1	27.6	24.4
Corugea	1738.7	781.2	351.0	351.0	350.0	351.0	89.4	69.6	53.6	36.2	36.2	23.8
Harsova	1349.6	620.1	284.9	130.7	130.7	89.9	75.4	75.4	45.9	31.3	31.3	20.9
Jurilovca	960.6	477.0	236.8	117.5	95.0	83.8	71.6	57.5	45.8	32.4	25.1	22.5
Mangalia	2032.7	900.6	399.0	176.5	137.9	119.3	99.2	76.9	59.0	39.5	29.3	25.8
Medgidia	734.9	394.3	211.5	113.3	93.8	84.0	72.9	60.0	49.0	36.1	28.7	26.1
Sulina	1272.7	563.7	249.6	110.4	86.2	74.6	62.0	48.1	36.9	24.7	18.3	16.2
Tulcea	1546.9	712.1	327.7	150.6	119.1	103.7	87.0	68.3	53.0	36.2	27.2	27.2

Table 8

Values (mm) corresponding to different exceedance probabilities, computed based on the Log Pearson III distribution function

Station	Probability of exceedance (%)											
	0.001	0.01	0.10	1	2	3	5	10	20	50	80	90
Adamclisi	393.9	270.7	181.0	115.7	99.8	91.0	80.7	67.5	55.0	38.8	28.6	24.9
Cernavoda	297.7	213.7	148.4	97.5	84.5	77.3	68.5	57.2	46.4	31.7	22.3	18.8
Constanta	943.0	533.0	293.4	154.6	125.7	110.9	94.1	74.3	57.3	37.5	26.9	23.3
Corugea	564.3	365.3	229.0	229.0	229.0	229.0	89.3	72.5	57.2	37.9	37.9	22.4
Harsova	196.5	164.5	131.9	98.4	98.4	81.7	73.7	73.7	50.9	33.5	33.5	18.1
Jurilovca	401.7	264.9	170.2	104.8	89.4	81.1	71.3	59.1	47.8	33.5	24.8	21.7
Mangalia	633.1	410.3	257.0	152.3	127.9	114.8	99.5	80.5	63.2	41.4	28.6	24.0
Medgidia	364.6	251.8	169.3	108.9	94.1	85.9	76.3	63.9	52.3	37.0	27.4	23.9
Sulina	386.3	252.1	158.9	94.8	79.7	71.6	62.1	50.3	39.5	25.9	17.9	15.0
Tulcea	490.0	327.0	211.3	129.4	109.8	99.2	86.7	71.0	56.5	37.9	26.6	26.6

Table 9

Values (mm) at different exceedance probabilities, computed utilizing the Weibull (L-moment) distribution function

Station	Probability of exceedance (%)											
	0.001	0.01	0.10	1	2	3	5	10	20	50	80	90
Adamclisi	110.0	102.0	92.6	80.7	76.4	73.62	69.8	63.9	56.6	42.6	29.1	22.6
Cernavoda	113.5	103.3	91.3	76.8	71.7	68.38	63.9	57.2	49.1	34.3	21.1	15.3
Constanta	157.8	142.4	124.6	103.3	95.8	91.08	84.7	75.0	63.5	43.0	25.5	18.0
Corugea	152.2	137.4	120.4	120.4	120.4	120.39	82.1	72.7	61.7	42.0	42.0	17.7
Jurilovca	109.3	100.3	89.8	76.8	72.1	69.11	65.0	58.8	51.2	37.0	23.9	17.9
Harsova	125.1	113.4	99.8	83.5	83.5	73.99	69.0	69.0	52.5	36.2	36.2	15.7
Mangalia	171.7	154.6	135.1	111.7	103.4	98.26	91.3	80.7	68.2	45.9	27.0	19.0
Medgidia	103.5	96.1	87.3	76.3	72.2	69.65	66.1	60.5	53.7	40.6	27.8	21.7
Sulina	107.4	96.7	84.5	69.8	64.7	61.44	57.1	50.4	42.6	28.7	16.9	11.9
Tulcea	144.1	130.6	115.0	96.2	89.6	85.34	79.6	70.9	60.6	41.8	25.4	25.4

For example, for the Constanta series, the correlation coefficient is between 0.84 (Weibull L-Moment distribution) and 0.93 (Pareto distribution). However, we consider the values obtained for the exceedance probability of 0.001% using EV2 Max L-Moment and Log Pearson III too high. For this reason and analyzing values of the determination coefficient, the Pareto distribution function has been kept for the next calculus.

Using the values from Table 6, the 24 h maximum annual precipitation series for each temporal exceedance probability ($h_{24h\ p\%T}$) have been represented in Fig. 10 as functions of the spatial probabilities (p%S). Further, the $h_{24h\ p\%T \cap p\%S}$ series have been computed and presented in Fig. 11. Most values are less than 200 mm, except for $h_{24hp\%T \cap p\%S}$ at 0.1% and 0.01%.

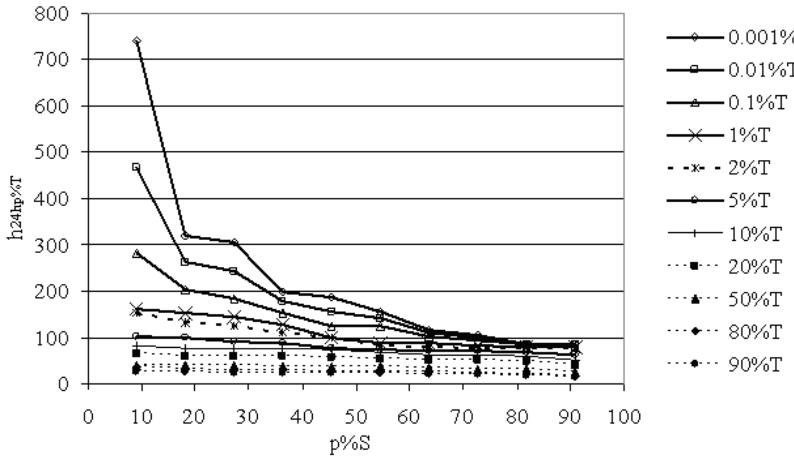

Fig. 10 – $h_{24hp\%T}$ chart based on the Pareto distribution for the set of series recorded at the ten stations for different exceedance probabilities.

As already mentioned, the studied surface is 15700 km^2. Any p%S value represents a percentage of this surface. For example, $h_{24h\ 10\%\cap 5\%S}$ = 85 mm is representative of a surface of 5% of 15700 km^2, that is 785 km^2, with a temporal probability of 10%. Considering the representative surfaces (F_B) equals to 10, 50, 100, 150, 250, and 500 km^2 and surfaces of 15700, 10000, 5000, 1000, 500, and 200 km^2 for F_Z, the spatial probabilities from Table 10 $p_{\%S}$, are obtained using the relation (1).

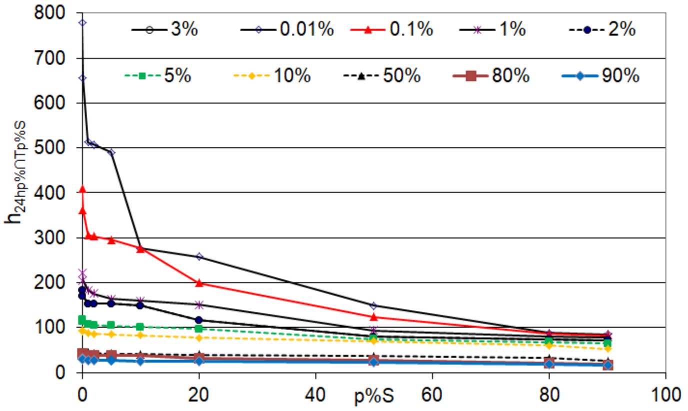

Fig. 11 – $h_{24hp\%T\cap p\%S}$ chart for the studied region.

Table 10

$p_{\%S} = f(F_Z, F_B)$

F_B (km^2) \ F_z (km^2)	200	500	1000	5000	10000	15700
10	5	2	1	0.2	0.1	0.06
50	25	10	5	1	0.5	0.3
100	50	20	10	2	1	0.6
150	75	30	15	3	1.5	0.9
250		50	25	5	2.5	1.6
500			50	10	5	3.2

Table 11 contains the values calculated utilizing the data from Table 9 and Fig. 11. Figure 12 is plotted based on the values from Table 11.

For the temporal probability of 1% or 5%, the curves are almost parallel in the zone between 12500 and 15700 km^2. For a standard surface of 15700 km^2, the 24 hour-maximum rainfall corresponding to a return period of 100 years on a surface of 10 km^2 is 212.1 mm. The 24 hour – maximum rainfall corresponding to a return period of 20 years over an area of 100 km^2 tends to be about 109.3 mm (the turquoise line in Table 10). The values are influenced by the distribution function selected. It is recommended to use a large F_z to achieve results close to the statistical estimation.

Table 11

The 24 hour maximum annual rainfall at p%T = 5% and 1% function of F_z and F_B

$p_{\%S}$	F_B	F_Z	$h_{24h5\%T}$	$h_{24h1\%T}$
0.06	10	15700	115.3	212.1
0.1	10	10000	113.18	204.80
0.2	10	5000	112.5	202.4
1	10	1000	106.71	183.60
2	10	500	106.06	175.70
5	10	200	104.12	163.51
10	10	100	101.20	159.57
20	10	50	97.10	150.68
50	10	20	73.97	93.59
0.3	50	15700	111.7	200.1
0.5	50	10000	110.3	195.4
1.0	50	5000	106.71	183.60
5.0	50	1000	104.12	163.51
10.0	50	500	101.20	159.6
25.0	50	200	95.0	141.2
50.0	50	100	73.97	93.6
0.64	100	15700	109.3	192.1
1.0	100	10000	106.71	183.60
2.0	100	5000	106.06	175.70
10.0	100	1000	104.12	159.6
20.0	100	500	101.20	150.7
50.0	100	200	97.10	93.6
3.2	500	15700	105.3	170.8
5.0	500	10000	104.12	163.51
10.0	500	5000	101.20	159.6
50.0	500	1000	73.97	93.6

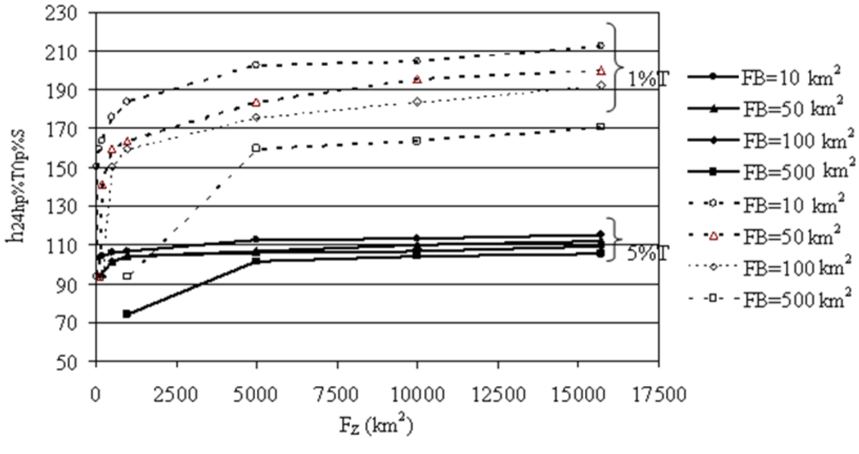

Fig. 12 – $h_{24hp\%T \cap p\%S}$ chart for the studied region.

3.3. COMPUTATION OF PMP_H AND COMPARISON WITH PMP_D

In the article [22], the PMP_H has been computed for the same series, using the methodology from Section 2.3 for determining k_{max} and replacing k_{max} with k_{avg} , the average of the values k_m computed for the ten stations. The results are summarized in Table 12.

The first remark is that PMP_H overestimates the 24 hour-maximum precipitation when computed based on formula (2) (the third column in Table 11). A significant improvement is noticed when k_{max} is replaced by k_{avg} for all the stations but Constanța for which the recorded value – 201 mm – is underestimated.

Table 12

PMP_H for the studied region

Location	Max	PMP_H with k_{max}	PMP_H with k_{avg}
Adamclisi	83.30	206.72	112.49
Cernavodă	73.60	202.89	106.52
Constanța	201.00	349.94	174.49
Corugea	114.80	278.91	143.65
Harșova	96.70	230.13	119.33
Jurilovca	79.90	208.66	110.32
Mangalia	146.00	322.69	164.74
Medgidia	84.60	195.78	106.65
Sulina	84.90	201.23	102.77
Tulcea	134.50	279.13	143.49

This method estimates the maximum precipitation at the recorded stations without considering the influence area of each station. Therefore it is a temporal method. By comparison, Diaconu's method provides a spatio-temporal evaluation of the 24 h probable maximum precipitation, based on the statistical distribution of the precipitation series and the spatial cover, giving a more realistic image of the PMP as a function of the return period and the area covered by the meteorological stations. Still, compared with the Hershfield method, the Diaconu one is more complicated and the accuracy depends on the fitted distribution. The last one is not an essential issue if the goodness of fit tests are performed to select the best distribution.

The main drawbacks of the Hershfield method are that the accuracy is significantly influenced by

- The number of series. The highest the number, the better the estimation is,
- The homogeneity of the data series: inhomogeneous data series produces inaccurate PMP_H estimation,
- The value of k_{max}, which is not always the best choice (as shown above) for the PMP_H computation.

4. CONCLUSION

In this article, we presented the spatio-temporal estimation of PMP based on Diaconu's method and a few comparisons with the classical Hershfield method. Both methods can be utilized depending on the user's purposes. For temporal purposes, PMP_H can be applied taking into account the above-specified issues, while for both temporal and spatio-temporal analysis, Diaconu's method is recommended.

Despite the easiness of use, of the Hershfield method, the choice of the parameter k that might replace k_{max} to improve the estimation quality remains an open problem.

REFERENCES

1. *Manual on Estimation of Probable Maximum Precipitation* (*PMP*), WMO-No. 1045; WMO: Geneva, Switzerland, 2009. https://library.wmo.int/doc_num.php?explnum_id=7706.
2. C. Clark and J. Dent, J. Geosci. Environ. Prot. **9**(7), 2009–2028 (2021).
3. J. Beauchamp, R. Leconte, M. Trudel, and F. Brissette, Water Resour. Resear. **49**, 3852–3862 (2013).
4. M. N. Desa, A. B. Noriah, and P. R. Rakhecha, Atmos. Res. **58**, 41 (2001).
5. E. M. Douglas, and A.P. Barros, J. Hydrometeorol. **4**, 1012–1024 (2003).
6. C. J. Wiesner, *Hydrometeorology*, London Chapman & Hall, 1970.
7. D. M. Hershfield, Proc. Am. Soc. Civil Eng., J. Hydraulics Div. **87**(*HY5*), 99 (1961).
8. D. M. Hershfield, J. Am. Waterworks Assoc. **57**, 965–972 (1965).
9. *Manual for estimation of probable maximum precipitation. Operational hydrology*, Rep.1, WMO – No. 332, WMO, Geneva, Switzerland 1986.
10. D. Koutsoyiannis, Water Resour. Res. **35**, 1313–1322 (1999).
11. D. Koutsoyiannis, Hydrol. Sci. J. **2004**, *49*(*4*), 591–610.
12. J. Duan, A. K. Sikka, and G. E. Grant, *A comparison of stochastic models for generating daily precipitation at the H.J. Andrews Experimental Forest*, Northwest Sci. **69**(4), 318–329 (1995).
13. A. Burgueño, M. D. Martínez, X. Lana, and C. Serra, Int. J. Climatol. **25**, 1381–1403 (2005).
14. G. M. Bonnin; D. Martin, B. Lin, T. Parzyok, M. Yekta, and D. Riley, *Precipitation-Frequency Atlas of the United States*, Vol. 1, Version 4.0: Semiarid Southwest, Arizona, Southeast California, Nevada, New Mexico (NOAA Atlas 14, 2006).
15. P. J. Pilon, K. Adamowski, and Y. Alila, Atmos. Resear. **27**, 81–92 (1991).
16. R. Deidda and M. Puliga, Phys. Chem. Earth **31**, 1240–1251 (2006).
17. C. Diaconu, Meteorol. Hydrol. **18**(2), Bucharest (1998) (in Romanian).
18. C. Diaconu, Rev. Hidrotehnica **35**, 385–399 (1990) (in Romanian).
19. C. Diaconu and P. Șerban, *Synthesis and Hydrological Zoning* (in Romanian), Editura Tehnică Bucharest, 1994.
20. S. Căzănescu and R. A. Căzănescu, Bull. Univ. Agri. Sci. Vet. Med. Cluj-Napoca, Agri. **66(2)**, 63–70 (2009).
21. A. Bărbulescu, Rom. Rep. Phys. **68**(2), 788–798 (2016).
22. A. Bărbulescu, C.Ș. Dumitriu, and C. Maftei, Rom. J. Phys. **67**, 801 (2022).
23. C. Ș. Dumitriu, *Assessing the fractal characteristics of precipitation series*, in *Scientific Researches*, Sevi Öz, Ed., IKSAD Publishing House, Ankara, 2021, pp. 95–114.
24. C. S. Dumitriu and A. Bărbulescu, *A New Tool for Determining the Irrigation Rate Based on Evapotranspiration Estimated by the Thornthwaite Equation*, Water **67**, 2399 (2022).
25. C. Maftei, A. Bărbulescu, S. Rugină, C. D. Năstac, and I. M. Dumitru, Water **13**, 374 (2021).
26. C. Diaconu, Rev. Hidrotehn. **33**(6), 210–215 (1988).
27. T. W. Anderson and D. A. Darling, Ann. Math. Stat. **23**, 193–212 (1952).
28. A. Bărbulescu, *Studies on Time Series. Applications in Environmental Sciences*, Springer Cham, 2016.