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Graphical Tool for Positioning of Triads in Mechanisms

Csaba Antonya*

Department of Automotive and Transport Engineering, Transilvania University of Brasov, Romania

*Corresponding author email: antonya@unitbv.ro

Abstract. The position of a triad (Assur group of class 3 and order 3) in a planar mechanism requires the computation of three parameters of the composing links. This can be obtained with analytical or graphical solutions. In this paper a graphical (or geometrical) solution is proposed, in which several 3D surfaces are generated with the geometrical parameters of the triad. The intersections of these surfaces will generate 3D curves and the solution of the problem will be the coordinates of a point at the intersection of a curve and a surface. This method can be applied regardless of the type of pairs in the triad. The advantage of the proposed geometrical method is that it offers a quick observable design of complex mechanisms in the analysis and synthesis process.

1. Introduction

Mechanism structural synthesis (called also number or topological synthesis) is the subject of several papers in the last decades. The synthesis aims to find suitable topologies for performing a task and fulfilling design constraints. Usually, this is performed by the aggregation of simpler kinematic chains. Modular topological structures are widely used in the field of mechanisms kinematic synthesis and analysis [1,2]. The kinematic synthesis deals with function generation, path generation, and of motion (or guidance) generation [3]. Position analysis can be obtained with analytical, numerical, or graphical tools. Usually, for obtaining the positions of the link, the kinematic loop equations are solved. Graphical methods are intuitive in the design process and provide quick access to visualisation of results, and also can distinguish singular and multiple solutions. The analysis and synthesis of topologies with a lower number of links are easy to perform, because most of the time they require two parameters. Planar graphical sketches can simply find two unknowns by intersecting circles and lines. In the case of three unknowns, the construction of 3D surfaces and curves is necessary and the intersection of these will lead to the solution of the problem.

The term triad is defined in [4] as a connected string of three vectors representing jointed rigid links of an actual mechanism. In this paper the original definition of the triad will be used as the Assur group of class 3 and order 3 [5]: a hinged structural group with a high number of links with three internal and external kinematic pairs.

2. Geometric Position Analysis of Mechanisms

Mechanism synthesis can be performed with a variety of approaches [6], most common procedures are based on closed and open kinematic chains. The closed chains are based on prototype mechanisms (for example the Stephenson and Watt planar 6-link complex kinematic chains [3]), each of the prototype mechanism will have revolute or prismatic pairs between the links and one of the link will be considered to be in a fixed position (the ground element). Other synthesis approaches are based on group theory techniques and exclusive generation of fractionated kinematic chains [6]. Some methods



need further analysis of the obtained mechanisms because they generate isomorphic chains or fractionated kinematic chains.

Assur groups are open chains and are an assembly of links and kinematic pairs with zero-mobility [7, 8]. These chains, if are added or removed from a linkage mechanism, do not change its mobility. For a mechanism there is a single decomposition into Assur groups. This concept is also used in topological graphs in the field of rigidity theory. The topology of the Assur groups can be found by using the Chebychev–Grübler–Kutzbach's equation for the mobility calculation of multi-loop mechanisms [9]. These are zero DOF mechanisms, so there are only a finite number of variations of structural groups with the corresponding number of kinematic pairs and links.

The simplest topologies of the Assur groups are the lower order monads and dyads (figure 1). These are one link with 1 DOF and 2 DOF external pair (the monads) and two links with three 1 DOF pairs (the dyad) with two external and one internal kinematic pairs. The most common higher-order Assur groups are the triads, tetrads, pentads. Triads (figure 2) are known also by the name of 6-bar linkage mechanism with high class structure groups [10] and 4-link Assur group [6]

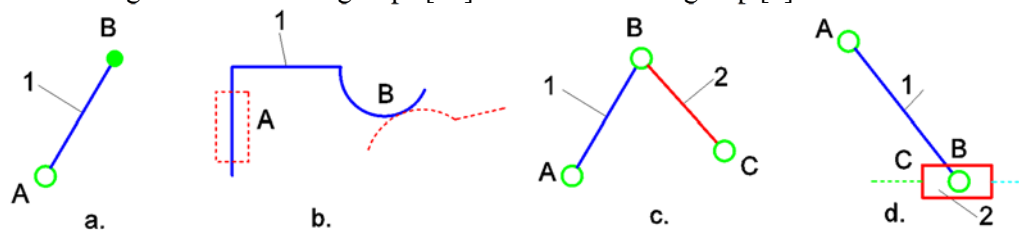


Figure 1. Monads (a, b) and dyads (c: RRR, d: RRP)

Analytical solutions for obtaining the positions of the Assur groups in mechanisms are using complex algebraic equations. These are the position equations obtained from closed vector equations or formulated with the relative motion constraints equations between the connected links [11,12]. These approaches will lead to a system non-linear position equation with the independent parameters describing the input links' movement and various numeric methods can be used for solving them.

A quick and easy alternative for the analytical method for obtaining the positions is the graphical or geometrical method. This simplify the problem and can be applied for all relevant configurations. Another advantage of the geometrical method is that it provides quick observable design and it can distinguish singular and multiple solutions. This can be transformed into geometric algebra because it provides the geometric sense of the algebraic expressions [13].

The positions of monads and dyads in a mechanism can be easily obtained with graphical tools. Because they are connected to the input link or the base, the unknown position is for only one pair (one external pair for the monad and the internal pair of the dyad). This means that two parameters are unknown (meaning two position coordinates). The geometrical tools for obtaining the two parameters are the intersection of lines or circles. Further, this can be transformed into analytical equations for analysis and position synthesis with different techniques (Gaussian relaxation technique, linear partition technique, genetic algorithms, differential evolution algorithms [14])

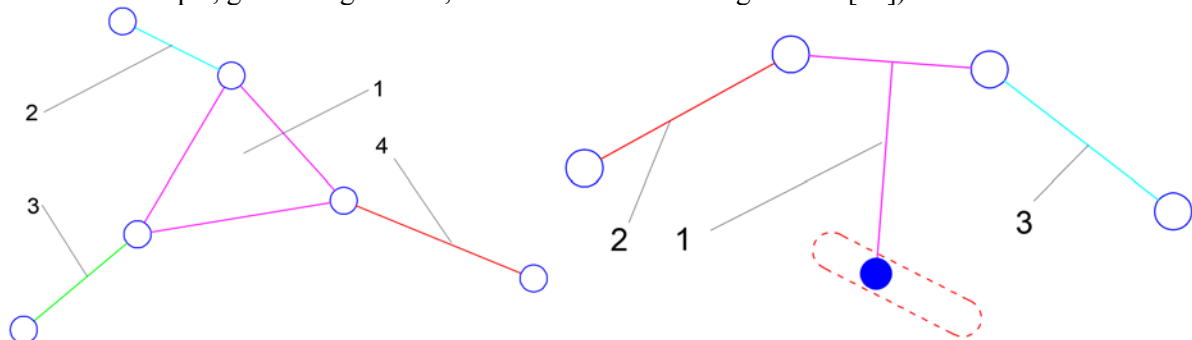


Figure 2. Triads with different topologies

The position of a triad cannot be found by planar geometrical tools, because it requires the solution for three unknowns (the position and orientation of one link). A procedure for the graphical synthesis of

the triad is proposed in [10]. It is based on finite rotation pole characteristic of the constitutive dyads to find the extreme positions of the links only, by simplifying the problem with a finite number of precise positions. To find the general solution, 3D geometry has to be used which will provide the coordinates of points of interest as the intersection between 3D surfaces and curves.

3. The Triad Positioning in Geometric Position Analysis

Usually, a triad is composed of the following links and pairs (figure 2):

- Three binary links with one DOF pairs at both ends and one ternary link with one DOF connections to the binary links.
- Two binary links with one DOF pairs at both ends and one ternary link with two 1 DOF connections to the binary links and one 2 DOF unpaired connection.

To form a one DOF mechanism, the unpaired connection will be attached to fix pivots (G and F) and one pivot will be attached to the driver link (2) of the mechanism (figure 3). The position of the triad is unique (or it can be a finite number of positions) and the coordinates of the ternary link (3) is defined by the length of the three binary link (4,5,6). This means that the three revolute pairs of link 3 are at known distances (EF, BC, DG) from three points (F, B, G).

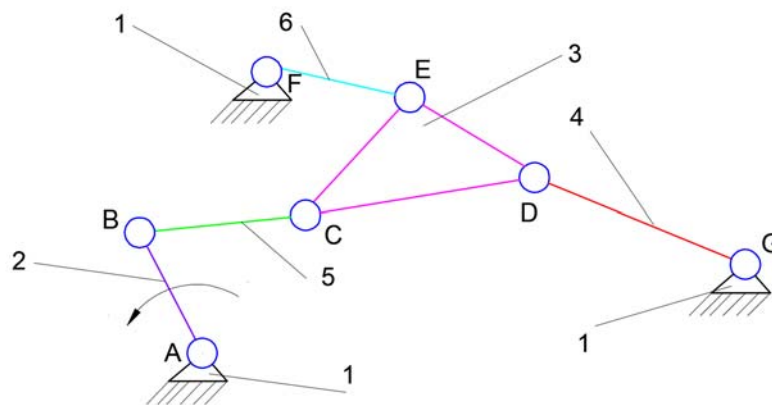


Figure 3. The triad in a mechanism

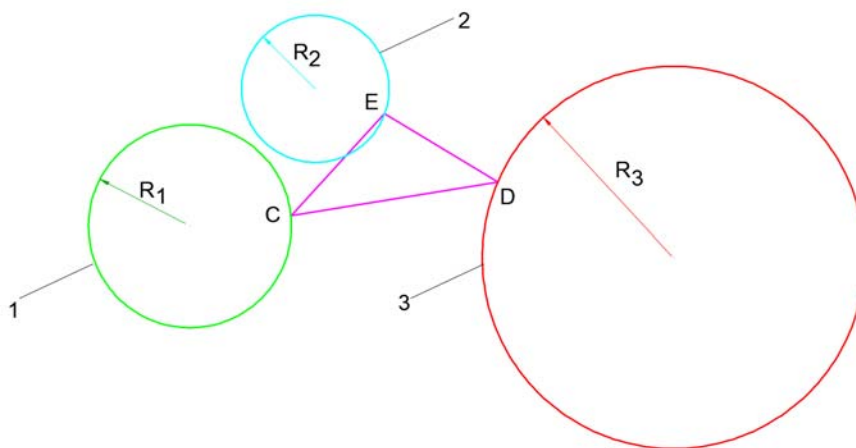


Figure 4. The triad positioning geometrical problem

To solve the problem of the triad positioning, the task can be reduced to the following geometrical problem (figure 4): find the position of given a scalene triangle with the vertices C, D, and E, knowing that these three points are belonging also to three circles (with known position) having radiuses R_1 , R_2 , R_3 ($R_1=BC$, $R_2=EF$, $R_3=GD$ – the length of the binary links of the triad).

The proposed solution requires the generation of five 3D surfaces. The steps of solving the problem are the following:

Step 1. Find the position of all points which are at distance CD from all the point on the circumference of the circle 1 (figure 4). For one point on the circumference this means a new circle with the radius

CD, but for all points they will belong to the surface of a 3D object (without the coordinate describing the height of the point). This object is generated by sweeping a circle with radius CD along a helix. The helix starting point can be randomly selected, the radius is R_1 , and the height can also be random (it is recommended to be considerable, to have better visual representation of the results). The swept circle has the radius CD (figure 5).

Step 2. Find the position of all points which are belonging to the circle 3 which will correspond to the points generated in step 1. This surface will be a cylinder with radius R_3 generated on top of circle 3 and the same height as the helix in step 1 (figure 5).

Step 3. Find all points which will satisfy both conditions from step 1 and step 2. These points will be on a 3D curve at the intersection of the two surfaces generated at the previous steps (C1 in figure 6).

Step 4. Repeat the procedure described in step 1 by sweeping a circle with radius CE along the same helix, generate the cylinder with radius R_2 with the same height on top of circle 2, and compute the intersection point of these two surfaces (C2 in figure 6).

Step 5. Find the coordinate of the points belonging to the previously obtained two 3D curves (C1 and C2) which are on the same height and are at the distance DE from each other. A new 3D object is generated by sweeping a circle of radius DE along one of the curves. The intersection of this surface with the other curve will satisfy the requirement and will be the solution to the problem (figure 7). This point will be projected to the plane and will represent one vertex of the triangle. The other two vertices (C and D) can be easily obtained by intersecting circles from this point (with radius EC and ED) and the other two circles (1 and 2).

The shape of the obtained two curves (C1 and C2) are depending on the starting point of the helix. Also, the height of the helix will influence the shape of the curves. The obtained point E will satisfy all requirements of the original problem.

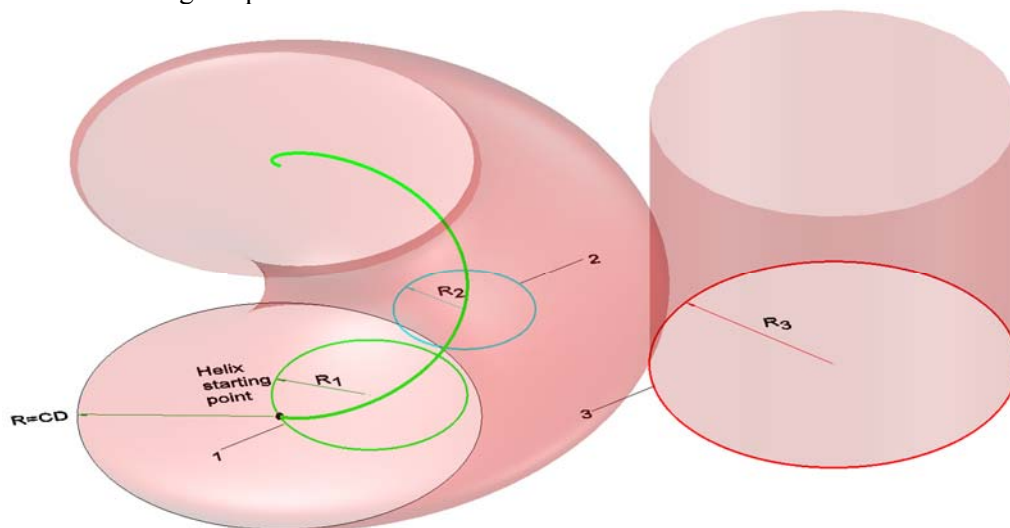


Figure 5. Intersection of two surfaces: a circle swept along a helix and a cylinder

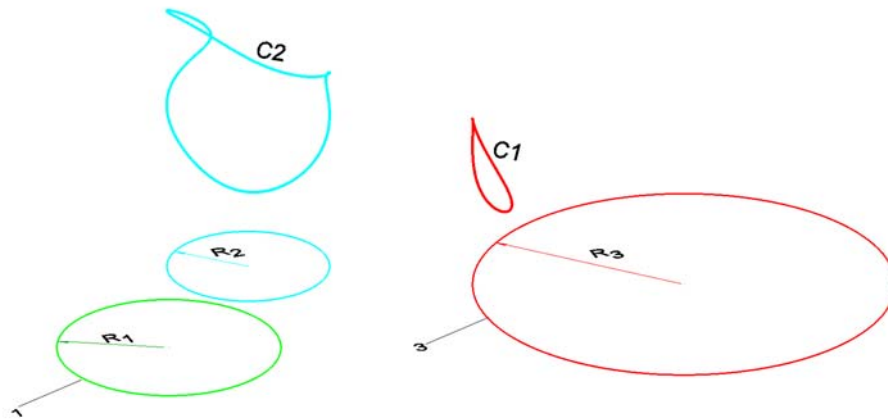


Figure 6. The 3D curves as the result of the intersection of the 3D surfaces

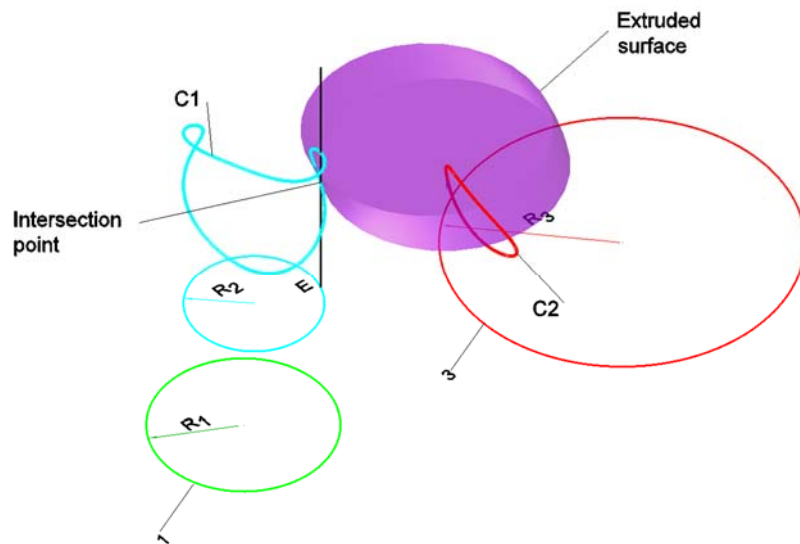


Figure 7. The intersection of the circle swept along one of the 3D curves and the other curve. This method can be applied also when the 3 points (C,D,E) are collinear following the same steps. In case the triad is connected to the ground link with a 2 DOF pair (figure 2), the intersection in step 2 will be not with a cylinder, but an extruded segment (planar surface). This segment is oriented along the permitted linear displacement along the slot.

A possible application of a mechanism with a triad is for a haptic tool for rehabilitation and training of the upper limb [15]. This device has to provide a mainly linear trajectory for the handle together with a small rotation in the vertical plane and force feedback in two directions. The proposed method is suitable for the graphical synthesis for finding the dimensions of the link to provide customized feedback force and trajectories, and is very valuable for analysing the performance of metallurgical equipment units. For example, to study the hinge-lever mould oscillator of the bow-type caster mould [16].

4. Conclusions

The proposed graphical tool aims to determine the configuration of a mechanism containing a triad for given values of the input link, in order to simulate its movement. This approach is different from previous works because it can be used for any type of triad and any relevant configuration. The triad positioning problem for planar mechanisms cannot be solved with 2D graphic tools (intersection of lines and circles) because it requires the computation of three parameters. The proposed method is constructing and intersecting 3D surfaces and curves. The surfaces are cylinders, planar surfaces, and a circle swept along a helix and a 3D curve. The intersection of surfaces can easily be transformed into

a system of mathematical equations. The method is suitable for analysis and synthesis of complex mechanisms and also offers quick observable design.

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