applied sciences

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The Quick Determination of a Fibrous Composite's Axial Young's Modulus via the FEM

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Featured Application: Determining Young's modulus is an important step in the analysis of any material that undergoes elastic deformation. A quick estimation of it can be performed using the finite element method (FEM) with the procedure presented in this paper.

Abstract: Knowing the mechanical properties of fiber-reinforced composite materials, which are currently widely used in various industrial branches, represents a major objective for designers. This happens when new materials are used that are not yet in production or for which the manufacturer cannot give values. Given the practical importance of this problem, several methods of determining these properties have been proposed, but most of them are laborious and require a long calculation time. And, some of the proposed calculation methods are very approximate, providing only upper and lower limits for these values. Experimental measurements are obviously the optimal solution for solving this problem, but it must be taken into account that this type of method consumes time and material resources. This paper proposes a sufficiently accurate and fast estimation method for determining Young's modulus for a homogenized fibrous material. Thus, the FEM is used to determine the natural frequencies of a standard bar, for which there are sufficiently precise classical methods to express the engineering constants according to the mechanical properties of the component phases of the homogenized material. In this paper, Young's modulus is determined for such a material using the relationships that provide the eigenfrequencies for the longitudinal vibrations. With the adopted model, transverse and torsional vibrations are eliminated by blocking the nodes on the surfaces of the bars. In this way, more longitudinal eigenfrequencies can be obtained, so the precision in calculating Young's modulus is increased.

Keywords: Young's modulus; longitudinal vibration; composite; FEM; fiber-reinforced material

Citation: Itu, C.; Scutaru, M.L.; Vlase, S. The Quick Determination of a Fibrous Composite's Axial Young's Modulus via the FEM. *Appl. Sci.* **2024**, *14*, 6630. https://doi.org/10.3390/app14156630

Academic Editor: Ana Martins Amaro

Received: 16 July 2024
Revised: 19 July 2024
Accepted: 19 July 2024
Published: 29 July 2024

1. Introduction

Determining Young's modulus for a material is a step during the design of a mechanical system. Its determination can be achieved experimentally or using different calculation formulas, methods for which there is a rich body of literature. For fibrous composite materials in particular, numerous methods have been developed to determine this value, with greater or lesser precision. The main methods developed are based on micromechanical models or homogenization theory. Their use involves determining the fields of strains and stresses for the respective system. Another method used by researchers, based on variational principles, generally provides upper and lower bounds for the modulus of elasticity. Obviously, in this case, the result is given by the difference between the upper and lower values of the modulus, and this method can lead, for certain concentrations of the composite phases, to errors, which are sometimes significant [1]. These limits were established in the cited paper for an orthotropic and a transversally isotropic composite. These methods are based on the consideration of particular cases of loading and boundary conditions and can present significant errors [2,3]. Micromechanical models are theoretically more accurate but require knowledge of the fields of stresses and strains for the material [4,5]. The results obtained in these studies have been developed and verified experimentally, and the results agree very well [6,7]. Fiber-reinforced composites represent an important class of materials with many applications in practice, and, as a result, numerous studies have been carried out on their mechanical properties [8–16]. These studies have generated numerous problems related to the methods of solving the equations obtained and the numerical results obtained [17–20]. Experimental methods are obviously the most reliable methods to determine the values of the elastic constants for a material. However, their use requires time and resources that can be significant in certain cases. This is why, in the first phase of a project, it is more advantageous to work with some estimates obtained quickly that have a satisfactory degree of confidence. Following this, in the final part of the project, more in-depth investigations should be carried out regarding the properties of the materials that have been decided to be used.

The method of a representative volume element for a composite material with short fibers was analyzed in [21]. The method can be applied in many applications, for example, in reinforced composites with aligned cylindrical fibers.

The finite element method (FEM) has become an established method in the analysis of deformable solids and has been successfully used by researchers to determine the mechanical behavior of composite materials. Generally, this method is applied for static analysis, mainly to determine the stress and strain fields inside a composite. In this paper, the FEM was used to study the vibrations of a composite and determine the natural frequencies of the considered specimen. The advantage of this method is that it can provide fast results for a very wide class of materials with different topologies, geometries, and mechanical properties. The ability of the FEM to obtain results very close to reality was demonstrated by experimental verification [22]. The FEM proved, once again, to be a useful method for studying the mechanical properties of composite materials made up of a polymer matrix reinforced with aligned cylindrical fibers [23]. It should be mentioned that most calculation methods involve the determination of the fields of stresses and strains for a certain load case (boundary conditions). Obviously, for almost all cases encountered in practice, this calculation involves a numerical method. For the determination of strains and stresses for a deformable solid, the method used with the greatest success at the moment is the FEM. As almost any analytical method proposed for the calculation of engineering constants involves the determination of the fields of strains and stresses, the FEM has become an important tool for the development of research in this field. In the current study, the FEM is used to analyze a very simple mechanical system, which can be modeled with well-known finite elements.

In engineering practice, situations may arise in which it is necessary to take into account other factors, such as temperature and humidity. In these cases, it is necessary that the classical models used also take into account these factors when obtaining some analytical relations that contain Young's modulus. The FEM can be used in classical models to obtain natural frequencies, but in classical models, additional factors that influence the mechanical behavior of the materials must also be considered. A very useful model for determining the elastic constants for a polymeric composite reinforced with unidirectional graphite fibers was developed in [24]. Developments of this model were undertaken in [25,26].

Obviously, the FEM can be used to determine elastic constants, alongside conducting a static analysis of the material using the generalized Hooke's law. But, this method implies, in order to obtain the appropriate precision, the consideration of several loading cases. Also, an averaging method must be used, for example, the least-squares method. But, this requires the use of the FEM in several stages. In the method presented in this paper, for the determination of Young's modulus, only one calculation of natural frequencies is necessary (the FEM is applied once). Moreover, several natural frequencies are obtained to calculate this quantity (any number of frequencies can be considered), and the obtained values can then be averaged, obtaining better accuracy. This saves time and costs.

For the determination of Young's modulus using experimental techniques that provide these values with high precision under laboratory conditions, some results are presented in [27], where Young's moduli were determined for brass, copper, Plexiglass, and PVC and illustrate the proposed experimental method. The material studied in the paper has a transverse isotropic behavior. Pipes for current applications are made of rubber that is reinforced with metal braids. Considering the complexity and the complicated manufacturing method, which introduces parameters that are difficult to control, for these types of materials, the best method for determining the mechanical properties is the experimental method. Thus, in [28], a new method for the experimental determination of Young's modulus was proposed. To achieve this, the Euler–Bernoulli model was used, and three vibration modes were determined experimentally. Based on the obtained results, Young's modulus was calculated. The experimental determination of Young's modulus for a special type of fiber-metal laminated beams is described in [29].

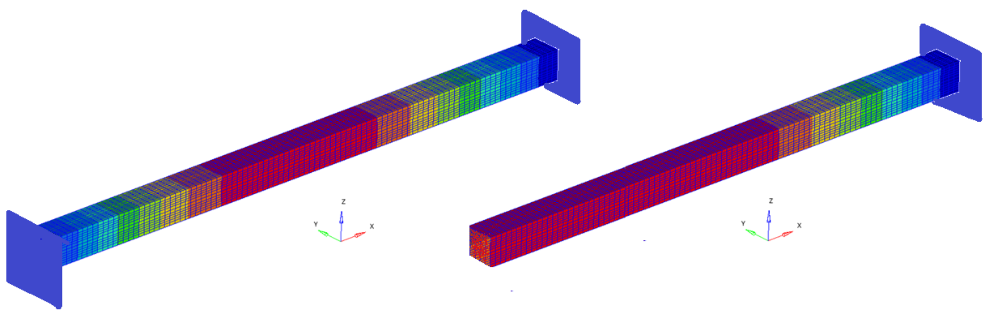
A classic model confirms the accuracy of the results obtained. The FEM is also used to determine the mechanical properties of some types of wood, a situation where there are insufficient data to describe the model and model-level approximations are needed [30]. Things become more complicated in such an analysis because wood is an orthotropic composite, so it is necessary to know more parameters to describe the constitutive law of the material. Once these values are determined, it is possible to study more complex models for practical situations that may be encountered in practice. In [31], such an approach was used to determine the mechanical properties of some species and types of wood material in order to use these values for design activities. Other experimental methods performed to determine the natural frequencies for transverse and torsional vibrations that allow the determination of some of the elastic constants are presented in [32] based on Timoshenko's theory. It is clear that, to determine the mechanical properties of composites, the best method, which has been intensively used by researchers, is to experimentally measure the eigenfrequencies of a material specimen and use these measurements to calculate the values of the engineering constants [33–36]. But, this method involves time and resources to perform the measurements, which is why it is desirable to use a quick method to determine properties in the first phase of design. Concrete is one of the materials that deserves special attention due to its extraordinary application in civil engineering. In this field, the current practice is to reinforce the concrete with cylindrical iron fibers. The FEM proved to also, in this case, be a suitable method for determining the mechanical properties of the homogenized material, and the elastic constants of the homogenized material were determined and verified experimentally [37]. A very good agreement was found. Another area where intense development is taking place is the use of natural fibers to reinforce a polymer matrix. However, studying these fibers is more complicated because they have a viscoelastic behavior [38], and the developed models are generally sophisticated. The analytical methods used are more elaborate and require more computational effort.

Another interesting method used in the study of composite materials reinforced with glass or carbon fibers, which exhibit a viscoelastic behavior, is presented in [39]. In developing the method, the time factor must also be introduced, which will describe the flow rate of the material if the temperature at which it is used increases. Other researchers have developed methods for determining elastic coefficients for different types of materials used in engineering applications [40,41]. A study on identifying material properties using an inverse method is presented in [42].

In this paper, the authors try to present the advantages of using the classical formulas from Vibration Mechanics to determine Young's modulus. A FEM model is used to calculate the natural frequencies of a beam. Two sets of boundary conditions are considered: the beam is clamped at both ends, and the beam is clamped at one of the ends. The material from which the beam is made is a polymer composite reinforced with carbon fibers. With the standard assumptions of the classical theory of straight bars, knowing the values of natural frequencies determined based on the formulas provided in the literature and knowing the values of the natural frequencies determined with the FEM model, Young's modulus can be determined. In this paper, the method is used to determine Young's modulus for a unidirectional carbon fiber-reinforced polymer composite material. The main advantage of this method is that it provides quick estimates of these values. Obviously, the costs involved are also reduced. If compared to experimental methods, the method is much cheaper and more convenient. Also, the time required to obtain these values is reduced. The analysis focuses on the study of longitudinal vibrations through conditions imposed on the displacements in the FEM, allowing only the axial displacement of the bar, considering a set of special boundary conditions. In this way, computing resources are used optimally. The method can be applied to a wide class of materials in order to determine the properties of the homogenized material.

2. Materials and Methods

The FEM is used to determine the natural frequencies of a composite material using all of the matrix material and fibrous material information. At the same time, the classical methods for determining the natural frequencies of a straight bar made of the same material, which represents a homogenized material whose properties are determined by the properties of the phases used, are considered. By comparing these values with those obtained using the FEM, the elastic constant values for the homogenized material can be obtained. To facilitate the presentation, some known results from the literature on straight-beam vibrations are presented very briefly. The classical assumptions from the theory of straight bars are considered fulfilled. We mention that we present only the important ones in the context of the paper [43]. Two cases are considered in our study: a beam clamped at both ends and a beam clamped at one end. Based on the relationships for determining the eigenfrequencies for a beam with the mentioned boundary conditions, it is possible to determine Young's modulus for the homogenized material. These values represent a quick estimate of Young's modulus. The FEM is used to determine these natural frequencies by considering the matrix and reinforcing fibers included in it. This can be accomplished relatively easily and accurately using this method. In this way, through a relatively simple calculation, considering the classical results obtained in the study of vibrations of straight bars to be valid, Young's modulus can be easily determined. Figure 1 shows the two types of analyzed beams. The software used is Hypermesh 2020. The type of finite element used is hexahedral.



(**a**) (**b**)

Figure 1. Beam (**a**) clamped at both ends; (**b**) clamped at one end.

Consider the longitudinal vibration of a straight beam. These vibrations are described by the following differential equations:

$$\frac{\partial^2 u}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 u}{\partial t^2} \quad (1)$$

If we choose a solution having the form

$$u(x,t) = \Phi(x)\sin(pt + \phi) \tag{2}$$

and introduce it into Equation (1), we obtain a differential equation that gives the amplitude of the natural vibrations (eigenfunctions):

$$\frac{d^2\Phi}{dx^2} + p^2\frac{\rho}{E}\Phi = 0 \tag{3}$$

The initial conditions, considered at the moment $t = 0$, are

$$u(x,0) = f(x); \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = g(x) \tag{4}$$

Equation (3) gives the solution:

$$\Phi(x) = C_1 \sin\alpha x + C_2 \cos\alpha x \tag{5}$$

where

$$\alpha^2 = p^2\frac{\rho}{E} \tag{6}$$

For a beam clamped at both ends, the boundary conditions are

$$\Phi(0) = 0; \;\; \Phi(l) = 0 \tag{7}$$

and it results easily in

$$C_2 = 0; \; C_1 \sin\alpha l = 0 \tag{8}$$

We easily obtain

$$\alpha = \frac{n\pi}{l}; \; n = 1,2,3,\ldots\ldots \tag{9}$$

Using notation (6), the eigenpulsations are given by the following relation:

$$p_n = \frac{n\pi}{l}\sqrt{\frac{E_L}{\rho}}; \; n = 1,2,3,\ldots\ldots \tag{10}$$

From Equation (10), the Young's modulus of the homogenized material clamped at both ends can be determined:

$$E_L = \frac{p_n^2\, l^2 \rho}{n^2\pi^2}; \; n = 1,2,3,\ldots\ldots \tag{11}$$

Using Equation (11) results in

$$p_1 = \frac{p_2}{2} = \frac{p_3}{3} = \ldots\ldots = \frac{p_n}{n} = ct; \; n = 1,2,3,\ldots\ldots \tag{12}$$

In a similar manner, if we consider the beam clamped only on one end, the literature [43] offers the following for the eigenpulsation:

$$p_n = \alpha_n\sqrt{\frac{E_L}{\rho}}; \; n = 0,2,3,\ldots\ldots \tag{13}$$

or

$$p_n = \frac{(2n-1)\pi}{2\,l}\sqrt{\frac{E_L}{\rho}}; \; n = 1,2,3,\ldots\ldots \tag{14}$$

So, the Young's modulus of the homogenized material clamped at one end can be determined with the following relation:

$$E_L = \frac{4p_n^2 l^2 \rho}{(2n-1)^2 \pi^2};\ n = 1, 2, 3, \ldots\ldots \quad (15)$$

Using Equation (14) results in

$$p_1 = \frac{p_2}{3} = \frac{p_3}{5} = \ldots\ldots = \frac{p_n}{2n-1} = ct;\ n = 1, 2, 3, \ldots\ldots \quad (16)$$

Using the FEM, it is possible to obtain a number of eigenpulsations that give us more values for E. The average can be calculated in order to obtain a better estimation of E.

3. Results

In this study, a carbon fiber-reinforced polymer was considered. With the help of finite element analysis, the material was modeled by taking into account the fibers embedded in it. The natural frequencies of the beam were determined using this model. Only the longitudinal vibrations of the material were considered. In the other two directions, the nodes are fixed (with these boundary conditions, we keep only the longitudinal vibration). This was implemented because when analyzing such a bar, the low modes are transverse, and torsional vibration modes and modes due to longitudinal vibrations occur at higher frequencies. This study aimed to obtain more frequencies for the longitudinal vibrations without having to identify and eliminate the other modes of vibration. This is achieved by blocking the movement of the beam surface in the other two directions. Given the classical model of the bar [43,44], the relationship between Young's modulus and the natural circular frequency can be written. If the natural circular frequency has been calculated with the FEM, then it is easy to obtain Young's modulus. This way of estimating Young's modulus has the advantage of simplicity. For the two bars with different boundary conditions, eigenfrequencies and eigenmodes were determined. Based on the values obtained in the calculation, the longitudinal Young's modulus was calculated [45–51].

The model considered in the calculations is shown in Figure 2 and is represented by a specimen with a square cross-section, with a length of 10 mm and a width of 1 mm, inside which 16 cylindrical carbon fibers aligned along the x-axis are embedded. The considered dimensions can be found in the figure. Young's modulus was considered to be 86.960 GPa for carbon fiber. For the polymer matrix, Young's modulus was considered to be 4.140 GPa. The density is 1850 kg/m^3 for the polymer matrix and 2000 kg/m^3 for the carbon fiber reinforcement. Poisson's ratio for the matrix was considered to be 0.22 and 0.34 for carbon fibers. Based on these data, the FEM model was built, with the help of which the eigenfrequencies were determined.

The density of the homogenized material, which is a necessary quantity for performing the calculations, is obtained with the following relation:

$$\rho = \nu_f \rho_f + \nu_m \rho_m. \quad (17)$$

Here, the fiber mass density is ρ_f, the matrix mass density is ρ_m, the fiber ratio is ν_f (fiber volume over total volume) and the matrix ratio is ν_m (matrix volume over total volume), ($\nu_m = 1 - \nu_f$).

The eigenmodes for the longitudinal vibrations for the first six eigenfrequencies are represented in Table 1. Based on the relationships that give us Young's modulus as a function of the eigenpulsation values, this quantity can now be determined. The results of the calculations are presented in Table 1. Thus, if the length of the beam specimen, the density of the homogenized material, and the natural frequency for a vibration mode determined with the FEM based on a simple model, which reproduces the two phases of the composite and its geometry, are known, Young's modulus can be obtained. It can be calculated for each frequency considered. It is reasonable to average the values obtained for several natural frequencies obtained by the FEM calculation. In this paper, we consider two boundary conditions for the bar specimen. First, the bar clamped at both ends is considered, and then the bar clamped at one end and free at the other end is analyzed.

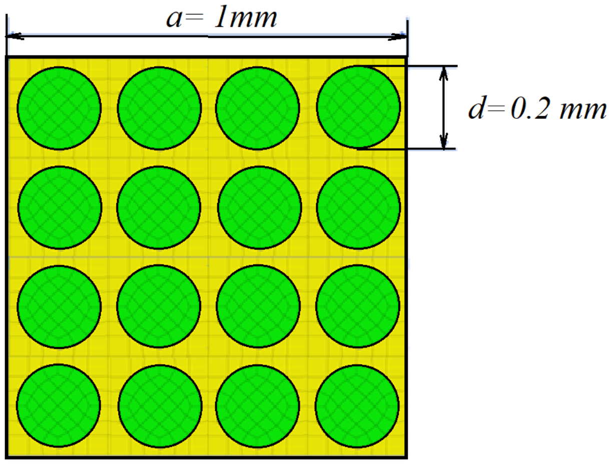


Figure 2. The composite specimen.

Table 1. Eigenpulsations obtained with FEM. Beam clamped at both ends.

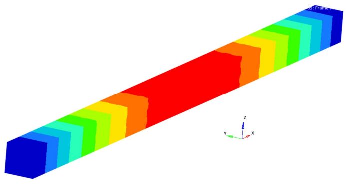
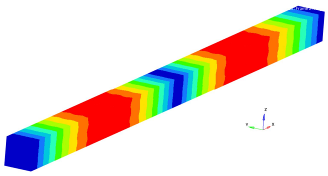
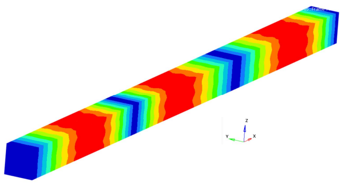
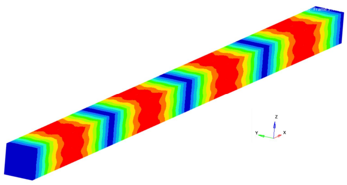
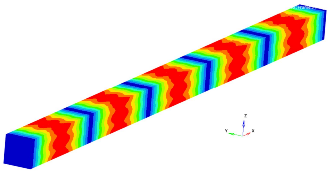
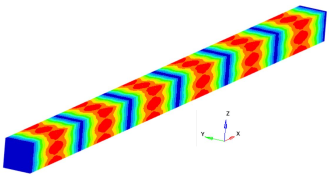
Mode No.	Eigenfrequency [Hz]	Representation	Ratio $\frac{p_n}{p_1}, n = \overline{1,6}$	Longitudinal Young's Modulus $E_L = \frac{4\nu_n^2 l^2 \rho}{n^2}$; E_L [GPa]
1	264,380.6		1.00	54.01
2	528,032.1		2.00	51.23
3	790,496.2		2.99	51.06
4	1,052,232		3.98	50.81

Table 1. *Cont.*

Mode No.	Eigenfrequency [Hz]	Representation	Ratio $\frac{p_n}{p_1}, n = \overline{1,6}$	Longitudinal Young's Modulus $E_L = \frac{4\nu_n^2 l^2 \rho}{n^2}$; E_L [GPa]
5	1,311,324		4.96	50.49
6	1,567,773		5.93	50.08
Average longitudinal Young's modulus E_L [GPa]				51.28

If it is taken into account that

$$p_n = 2\pi\nu_n \tag{18}$$

where ν_n is the natural frequency, the expression for the modulus of elasticity is obtained [47]:

$$E_L = \frac{4\nu_n^2 l^2 \rho}{n^2};\ n = 1, 2, 3, \ldots\ldots. \tag{19}$$

if the bar is clamped at both ends and

$$E_L = \frac{16\nu_n^2 l^2 \rho}{(2n-1)^2};\ n = 1, 2, 3, \ldots\ldots \tag{20}$$

if the bar is clamped at one end (Table 2).

Table 2. Eigenpulsations obtained with FEM. Beam clamped at one end.

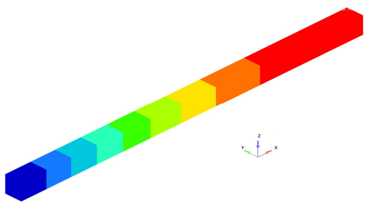
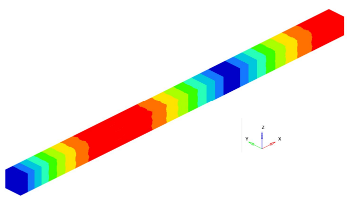
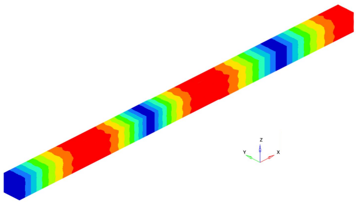
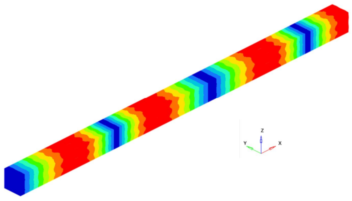
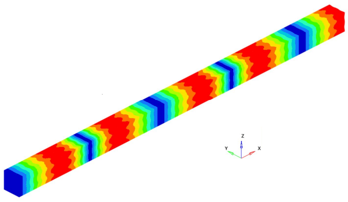
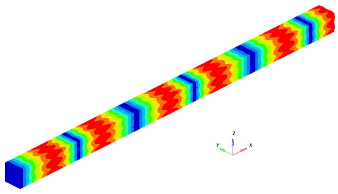
Mode No.	Eigenfrequency [Hz]	Representation	Ratio $\frac{p_n}{p_1}, n = \overline{1,6}$	Longitudinal Young's Modulus $E_L = \frac{16\nu_n^2 l^2 \rho}{(2n-1)^2}$; [GPa]
1	128,773		1.00	53.92
2	386,064		3.00	53.85

Table 2. *Cont.*

Mode No.	**Eigenfrequency [Hz]**	**Representation**	**Ratio** $\frac{p_n}{p_1}$, $n = \overline{1,6}$	**Longitudinal Young's Modulus** $E_L = \frac{16\nu_n^2 l^2 \rho}{(2n-1)^2}$; **[GPa]**
3	642,587		4.99	53.71
4	897,798		6.97	53.49
5	1,151,110		8.94	53.19
6	1,401,865		10.91	52.81
Average longitudinal Young's modulus E_L [GPa]				53.50

It is found that the values obtained for the bar clamped at one end are closer to each other than in the first case, when the bar is clamped at both ends.

A representation of the eigenfrequencies for the beam clamped at both ends is given in Figure 3.

The eigenfrequencies for the beam clamped at one end are represented in Figure 4.

The law of mixtures gives us the value 54.950 GPa.

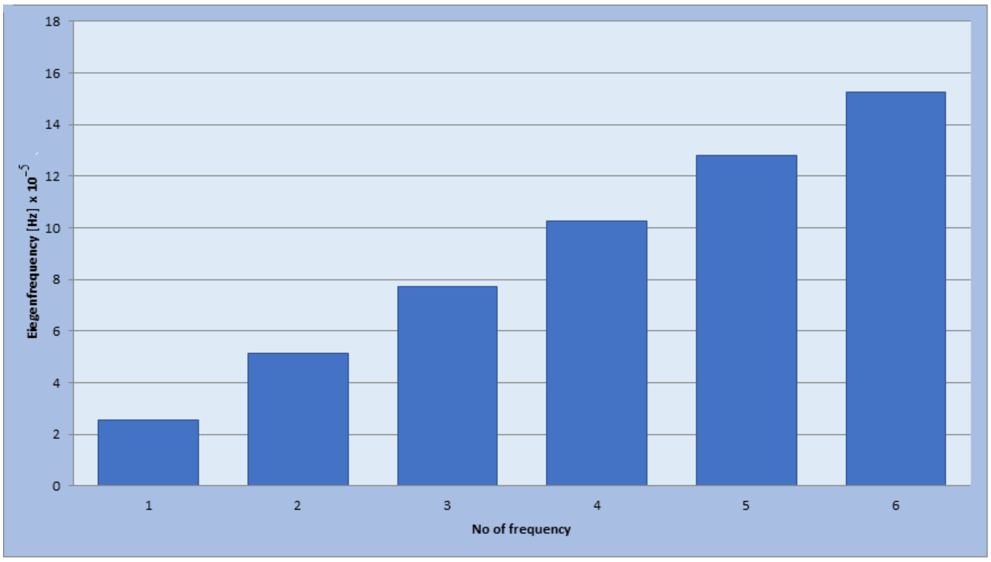


Figure 3. The eigenfrequencies for the beam clamped at both ends.

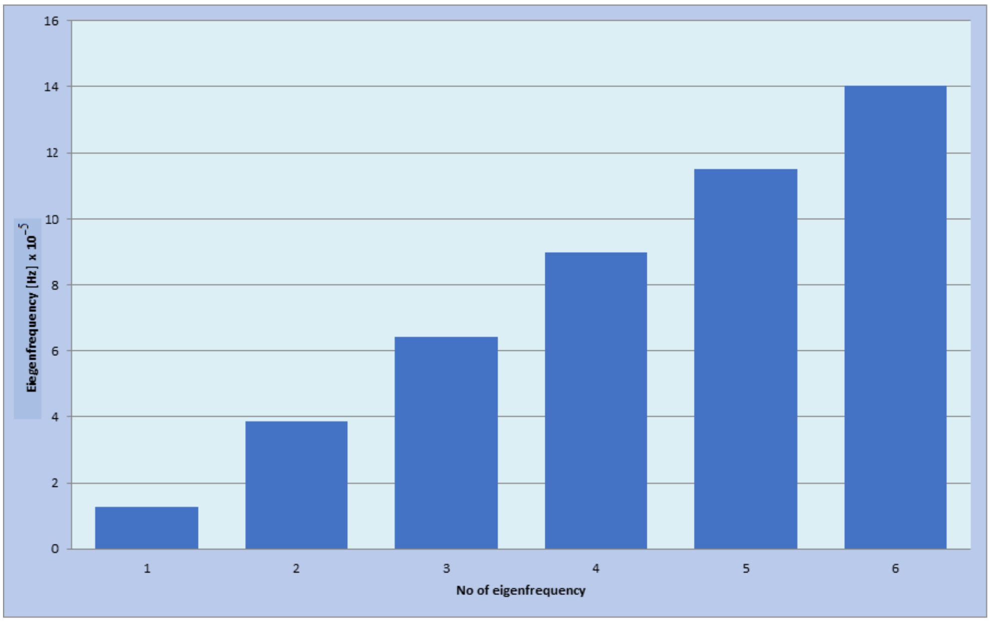


Figure 4. The eigenfrequencies for the beam clamped at one end.

4. Discussion

The values of the elastic constants that define the constitutive law of a composite made of a polymer matrix reinforced with glass or carbon fibers can be determined by several analytical methods developed by researchers, which are, however, very laborious. In most of the proposed methods, it is necessary to determine strain and stress fields for a specimen considered representative of the material. Boundary methods, which use variational calculations, impose restrictive boundary conditions that make the obtained results less accurate. Experimental methods remain the most reliable methods but have the disadvantage of the time in which they can be carried out and the resources involved (and thus costs). This paper shows that the FEM can be a simple and sufficiently accurate method for the quick estimation of some of the elastic constants. In this work, we dealt with the modulus of elasticity, but it is obvious that other constants can be determined if suitable models are used. In this paper, with the help of simple formulas, we managed to calculate Young's modulus. We can conclude that the presented method has the advantage of speed, simplicity and feasibility for obtaining quick estimates of the values of elastic constants in the design phases of a mechanical system. The accuracy of the results can be improved by considering a FEM model with more elements, which will more accurately describe the existing situation for each practical application [49–51]. This method provides fast and accurate estimates of Young's moduli, which are important data in the design process. Obviously, the assumptions for bars in the theory of straight bars must be respected. The errors that may appear are due to the FEM, within the limits accepted for the approximation method, and the deviation from classical assumptions. The method allows for obtaining quick and useful results. Obviously, the need to know the mechanical properties expressed by the elastic constants of the materials is a main objective when designing a mechanical system. Among these materials, polymer composites reinforced with aligned cylindrical fibers are distinguished, mainly due to their numerous practical applications. Obviously, experimental measurements provide the most reliable results, but this operation is expensive and time-consuming. This is why fast methods for estimating these properties according to the properties of the component phases are necessary. Based on micromechanical models, analytical formulas were determined to obtain engineering constants. But, this approach requires the determination of the stress and strain fields for the studied system. The same requirement is implied if homogenization theory is used. Other theoretical models have been proposed that involve the knowledge of particular loading situations but that provide upper and lower bounds for the elastic constants, which means significant errors in the determination of these constants, which can sometimes be far from the real values [7–13]. The method used in the present paper considers the homogenized material and uses classical calculation methods to determine the natural frequencies of such a homogenized bar. With the FEM, the values of the natural frequencies are determined for the real system, consisting of the matrix material and the reinforcing fibers. By comparing the two values, Young's modulus can be easily determined. In this way, fast and accurate estimates of this quantity are obtained [52,53]. In practice, complex situations may arise, determined by practical applications, in which effects that cannot be neglected, such as thermal effects, humidity, etc., may occur. In order to solve such problems, classical models must be improved to be able to take into account these effects, which introduce new parameters.

5. Conclusions

The method proposed in the present study provides a rapid estimation of Young's modulus for a fiber-reinforced composite material. The method can be useful for providing designers with the necessary data for the design process. In this method, the FEM is used, which is a calculation method that is well mastered by designers and provides accurate results. For the analyzed specimen, the eigenfrequencies are determined, which are then used to obtain, through simple calculations, the elastic modulus. It is necessary to know the properties of the phases that make up the composite. The FEM can be used to analyze static cases and thus determine the engineering constants using Hooke's law. To obtain values with sufficient accuracy, it is necessary to consider several load cases (as many as possible) and then use the least-squares method to determine the sought values. Such a method involves several work sessions and, therefore, the use of the FEM several times. The method proposed in this paper requires a single run of the program, which, however, allows several estimates of Young's modulus. Using all natural frequencies, a Young's modulus value is obtained, finally, by averaging. This leads to savings in time and resources. The simplicity and speed with which this method can be applied justify its use in the design phase. The estimation is mainly affected by the approximations and assumptions that are made for the classical straight beam and by the errors of the FEM, which is essentially an approximation method. In the example studied in this work, the results are similar to those obtained by other methods and then verified experimentally [48–50]. In real life, there are situations where the materials used work in special conditions of temperature, humidity, etc. The presented method can also be applied in these cases, working with models that include additional factors such as those listed above. There are therefore possibilities for research for these cases as well.

Author Contributions: Conceptualization, C.I., M.L.S. and S.V.; methodology, C.I., M.L.S. and S.V.; software, C.I.; validation, C.I., M.L.S. and S.V.; formal analysis, C.I., M.L.S. and S.V.; investigation, S.V.; resources, C.I., M.L.S. and S.V.; data curation, C.I.; writing—original draft preparation, S.V.; writing—review and editing, S.V.; visualization, C.I., M.L.S. and S.V.; supervision, C.I., M.L.S. and S.V.; project administration, M.L.S.; funding acquisition, C.I., M.L.S. and S.V. All authors have read and agreed to the published version of the manuscript.

Funding: The APC was funded by Transilvania University of Brasov, HBS 7302.

Institutional Review Board Statement: Not applicable.

Informed Consent Statement: Not applicable.

Data Availability Statement: The original contributions presented in the study are included in the article, further inquiries can be directed to the corresponding authors.

Conflicts of Interest: The authors declare no conflicts of interest.

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