Comparative dynamic analysis of two parallel robots

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Abstract. The paper presents a comparative dynamic modeling and VR (virtual reality) simulation for two 3 DOF medical parallel robots: a three coupled motions structure (Orthoglide robot) versus two coupled motion parallel structure (robot of type 1PRRR+2PRPaR). Kinematical and dynamical models, followed by a VR application with control aspects are presented for these two parallel robots. The innovative user interface for high-level control of the two parallel robots, presented in the paper, was developed in MATLAB - Simulink and SimMechanics environment, while the closed form dynamic models were obtained in MAPLE program. This kind of parallel robots can be successfully applied for medical applications where accuracy and high dynamic behavior are required. This research will lay a good foundation for the development of medical parallel robots.

Introduction

Over the last couple of decades parallel robots have been increasingly studied and developed from both theoretical viewpoints as well as for practical applications. Advances in computer technology and development of sophisticated control techniques have allowed for more recent practical implementation of parallel manipulators. Some of the advantages offered by parallel manipulators, when properly designed, include an excellent load-to-weight ratio, high stiffness and positioning accuracy and good dynamic behavior ([2], [3]). The ever-increasing number of publications dedicated to parallel robots illustrated very well this trend in research.

The parallel robots are mechanisms with closed kinematics chains, composed by an end-effector (the mobile platform) with n degrees of freedom, connected to the base by two or more kinematical chains called limbs or legs. A simple or a complex kinematics chain can be associated with each limb [2].

In this paper, a kinematics and a dynamic modeling are presented for two parallel robots with two and three degrees of coupling.

Different methods can be applied to dynamic modeling of parallel robots. In this paper, the Lagrange method with multipliers is used to derive the closed form dynamic model in the hypothesis of rigid links. Based on a representative numerical example, these results are verified using the CAD model implemented in ADAMS simulation environment.

The Virtual Reality (VR) immerses the user in a three-dimensional (3D) environment that can be actively interacted and explored. Virtual reality environment tool is used by many researchers in design, development and manufacturing of the robotic industry [4].

Using the virtual reality simulation with a virtual robot, a three dimensional design and the real-time behavior of the robot can be observed; that fact is relatively new and allows testing the robot before accomplishment a physical implementation. In this way, resources (money and time) can be saved and various problems can be solved from the design stage.

This paper presents the necessary steps for developing the virtual environment for kinematic simulation, starting from the SolidWorks model of these two parallel robots.

3PRPaR and 2PRRR+1PRPaR Parallel Robots Description

The first parallel robot studied is Orthoglide, a 3 DOF parallel robot ([1], [5]) with three coupled motions of type 3PRPaR (3 – Prismatic – Revolute – Parallelogram - Revolute), composed by a mobile platform connected to the fixed frame by three kinematics chains (Fig. 1).

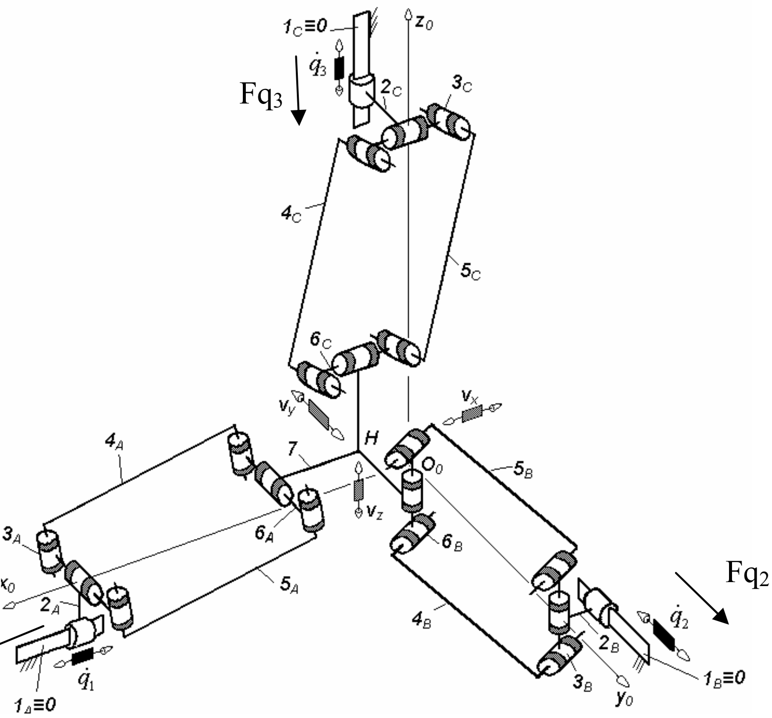

Fig. 1. Kinematic structure of Orthoglide parallel robot also known as 3PRPaR parallel robot [1].

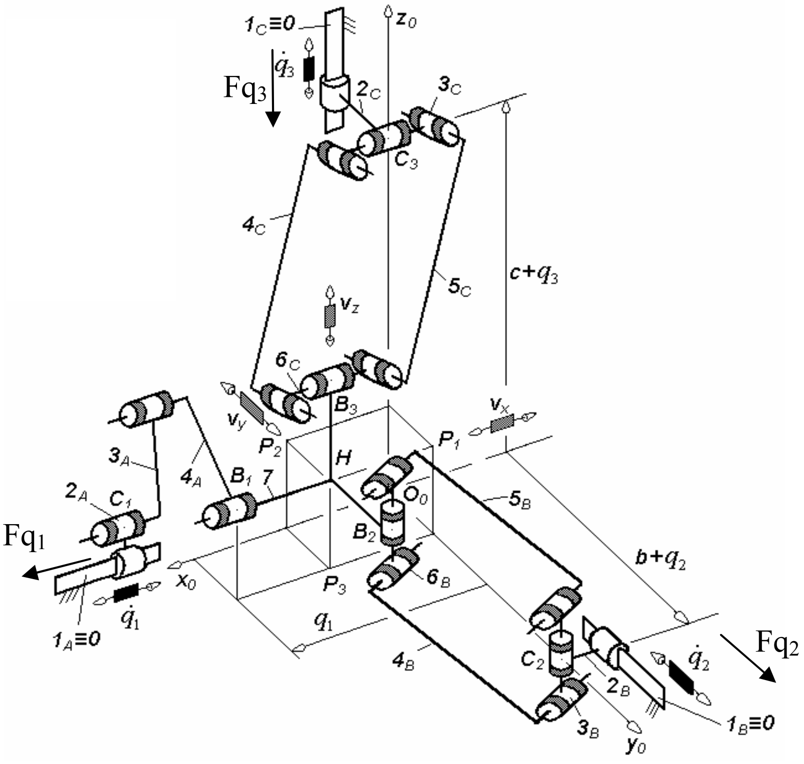

Fig. 2. Kinematic structure of parallel robot of type 1PRRR+2PRPaR [1].

The second parallel robot studied is a 3DOF parallel robot [1], [2] with one coupled motions of type 1PRRR+2PRPaR (1 – Prismatic – Revolute – Revolute – Revolute and

2 – Prismatic – Revolute – Parallelogram – Revolute), composed by a mobile platform connected to the fixed frame by three kinematics chains (Fig. 2).

Kinematics Modeling Of the 3 DOF Parallel Robots and Virtual Reality Tool

These parts of paper present the kinematics model for the two parallel robots. Typically, the study of the robot kinematics is divided into two parts, forward (or direct) kinematics and inverse kinematics.

The forward kinematics problem involves the mapping from a known set of input joint variables to a pose of the moving platform that results from those given inputs. However, the inverse and forward kinematics problems of our parallel robots can be described in closed form.

The inverse kinematics problem involves a known pose (position and orientation) of the output platform of the parallel robot to a set of input joint variables that will achieve that pose.

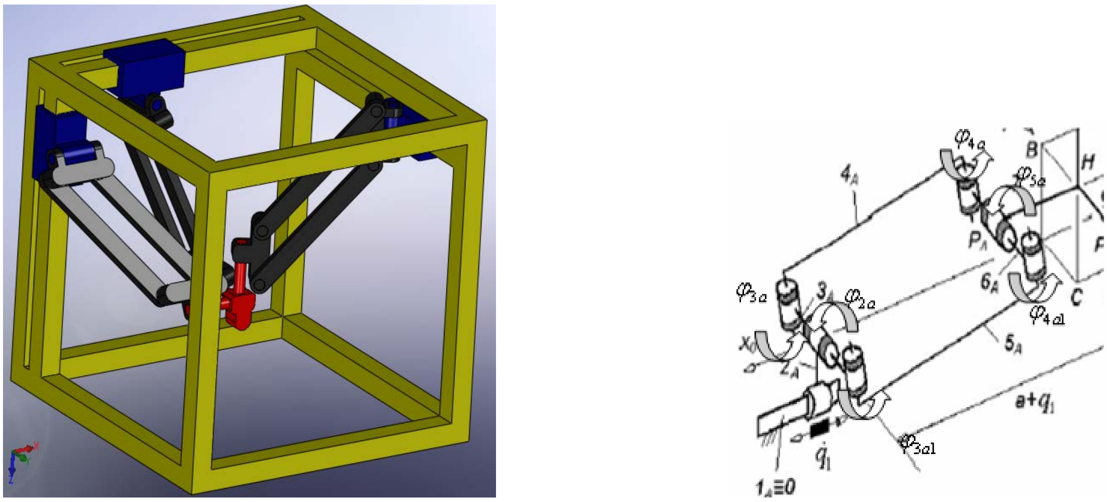

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Fig. 3. The CAD model of Orthoglide parallel robot (a) and the notations for non-actuated (passive) joints (b).

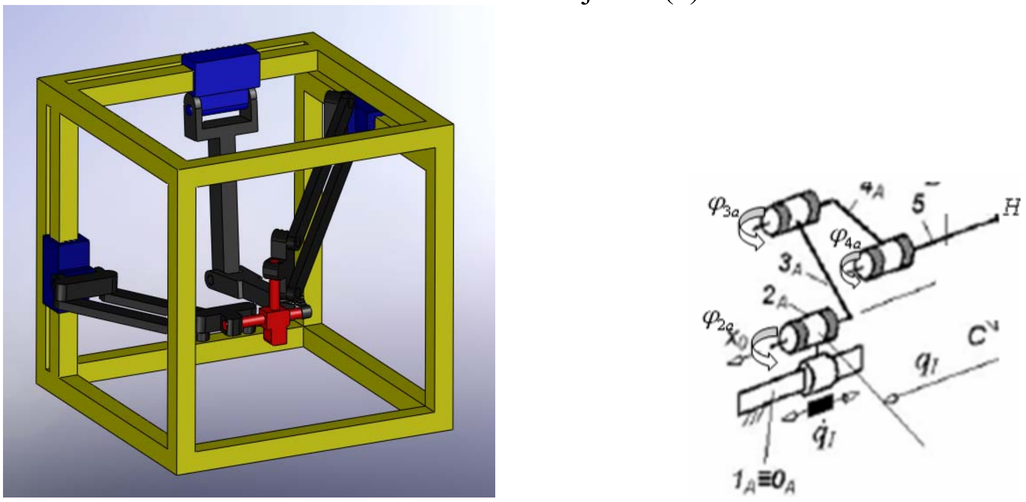

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Fig. 4. The CAD model of parallel robot de type 1PRRR+2PRPaR (a) and the notations for passive joints (for the uncoupled motion links) (b).

A. Direct and inverse kinematics model of Orthoglide parallel robot and of11PRRR+2PRPaR parallel robot

The direct kinematical model (Eq. 1) determines the mobile platform velocity (V_{xh}, V_{yh}, V_{zh}) in function of drivers' motion ($\dot{q}_1, \dot{q}_2, \dot{q}_3$).

$$\begin{bmatrix} v_{xh} \\ v_{yh} \\ v_{zh} \end{bmatrix} = J_h \cdot \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}, \tag{1}$$

where: J_h is the Jacobian of the system.

The inverse model (geometrical and kinematical) permit to obtain the liaison between $\{q_1, q_2, q_3\}$ and $\{x_h, y_h, z_h\}$ - the inverse geometrical model – eq. 2 for 1PRRR+2PRPaR parallel robot (with notations presented in fig. 1-4):

$$\begin{cases} q_1 = x_h + l_{7a} \\ q_2 = \sqrt{l_{4b}^2 - z_h^2 - x_h^2} + l_{7b} + y_h \\ q_3 = \sqrt{l_{4c}^2 - x_h^2 - y_h^2} + l_{7c} + z_h \end{cases} \tag{2}$$

and eq. 3 for Orthoglide parallel robot (with notations presented in fig. 1-4):

$$\begin{cases} q_1 = \sqrt{l_{4a}^2 - y_h^2 - z_h^2} + l_{7a} + x_h \\ q_2 = \sqrt{l_{4b}^2 - z_h^2 - x_h^2} + l_{7b} + y_h \\ q_3 = \sqrt{l_{4c}^2 - x_h^2 - y_h^2} + l_{7c} + z_h \end{cases} \tag{3}$$

Deriving the rel. 2 and 3 the inverse kinematical model (eq. 4) is obtained:

$$\begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix} = J_h^{-1} \cdot \begin{bmatrix} v_{xh} \\ v_{yh} \\ v_{zh} \end{bmatrix}. \tag{4}$$

Where J_h^{-1} is inverse of Jacobian matrix and the expression of this matrix is presented in eq. (5) for or 1PRRR+2PRPaR parallel robot:

$$J_h^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -\dfrac{x_h}{\sqrt{l_{4b}^2 - z_h^2 - x_h^2}} & 1 & -\dfrac{z_h}{\sqrt{l_{4b}^2 - z_h^2 - x_h^2}} \\ -\dfrac{x_h}{\sqrt{l_{4c}^2 - x_h^2 - y_h^2}} & -\dfrac{y_h}{\sqrt{l_{4c}^2 - x_h^2 - y_h^2}} & 1 \end{bmatrix} \tag{5}$$

and in eq. 6 for Orthoglide parallel robot:

$$J_h^{-1} = \begin{bmatrix} 1 & -\frac{y_h}{\sqrt{l_{4a}^2 - y_h^2 - z_h^2}} & -\frac{z_h}{\sqrt{l_{4a}^2 - y_h^2 - z_h^2}} \\ -\frac{x_h}{\sqrt{l_{4b}^2 - z_h^2 - x_h^2}} & 1 & -\frac{z_h}{\sqrt{l_{4b}^2 - z_h^2 - x_h^2}} \\ -\frac{x_h}{\sqrt{l_{4c}^2 - x_h^2 - y_h^2}} & -\frac{y_h}{\sqrt{l_{4c}^2 - x_h^2 - y_h^2}} & 1 \end{bmatrix} . \tag{6}$$

B. Virtual Reality tools of parallel robots

An innovative user interface for high-level control of robot manipulators is presented in this section. The interface is based on a virtual reality approach in order to provide the user with an interactive 3D graphical representation of the parallel robot. The interface was designed to give to a novice user an intuitive tool to control any kind of mechanical structure (serial, parallel or hybrid), requiring no programming skills. Computer based simulation allows mimicking of a real life or potential situations. SimMechanics models, however, can be interfaced seamlessly with ordinary Simulink block diagrams.

The actuators and the control algorithm were modeled with Simulink. The dynamic model of the mechanical structure was imported from SolidWorks using SimMechanics from MATLAB/Simulink (Fig. 5).

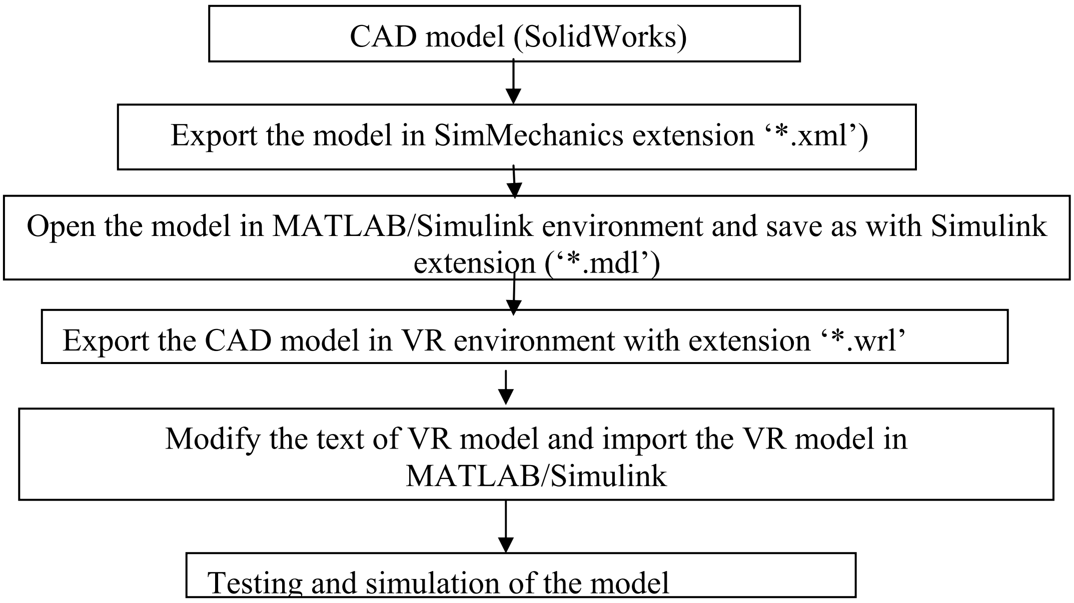

Fig. 5. The algorithm for the VR model obtained from the SolidWorks CAD model of the robot.

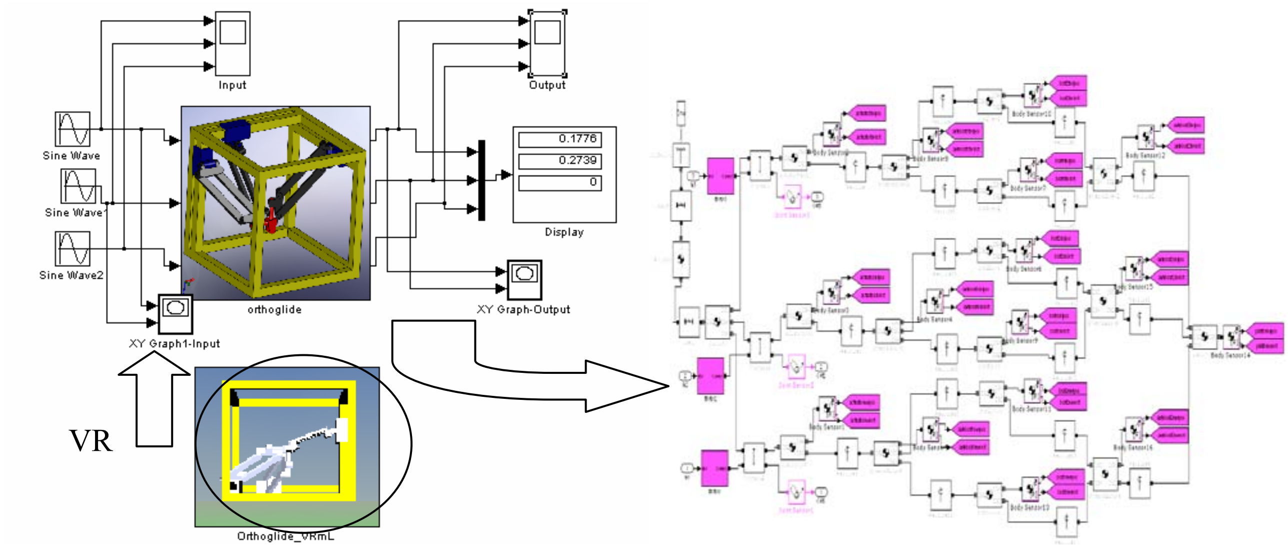

Fig. 6. The complete model for Orthoglide parallel robot with VR.

In figure 6 the complete model with VR (developed in fig. 7) of Orthoglide parallel robot is presented.

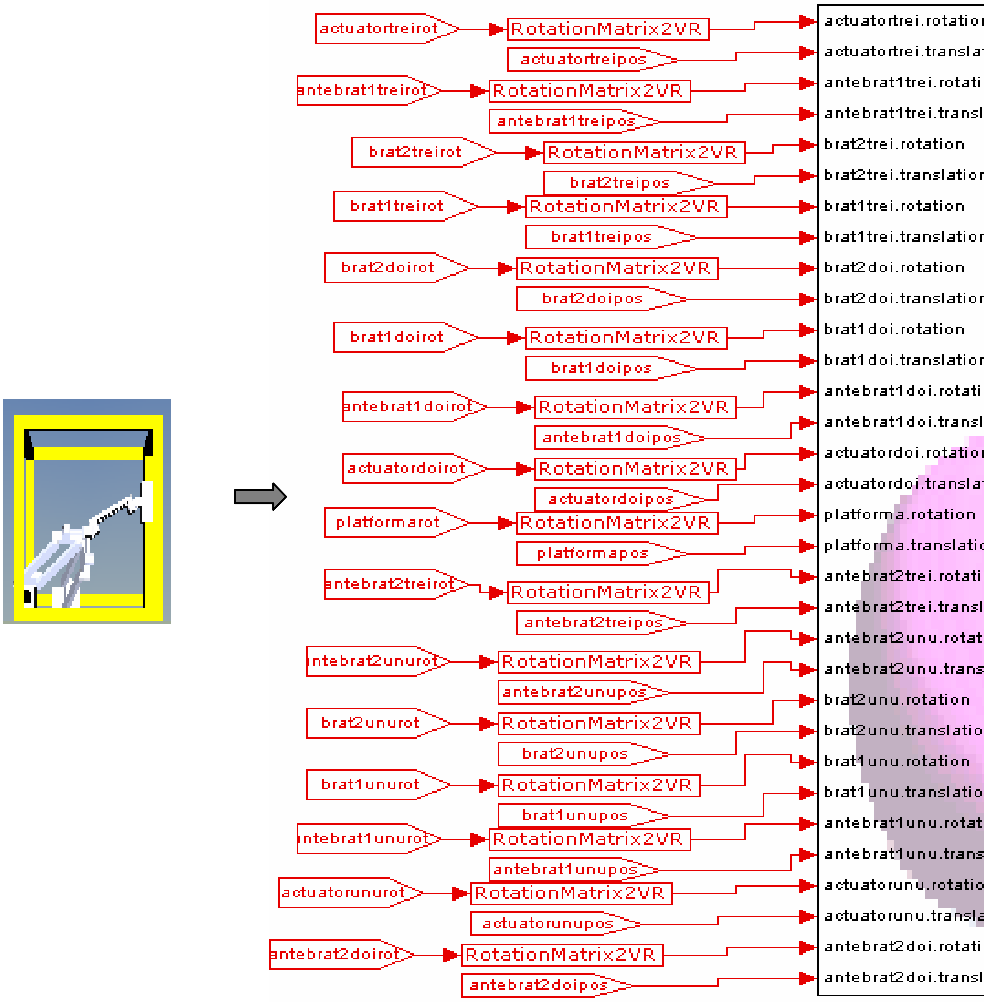

Fig. 7 Virtual Reality tool for Orthoglide parallel robot.

Virtual Reality Toolbox for MATLAB makes more realistic renderings of bodies possible. Arbitrary virtual worlds can be designed with Virtual Reality Modeling Language (VRML), and interfaced to the SimMechanics model. The user simply describes the geometric properties of the robot first. Then, in order to move any part of the robot through 3D input devices, the inverse kinematics is automatically calculated in real time. The interface was also designed to provide the user decision capabilities when problems such as singularities are encountered.

VR interface enables users to interact with the robot in an intuitive way. This means that the operator can pick and choose any part of the robot and move it using translation and rotation using 3D sensors, as easily as a "drag and drop" operation is. Thus, trajectories can be easily defined, optimized, and stored. Not only that, but the virtual world can be accessed and controlled via the Internet, too.

The use of a VR interface to simulate robots drastically improves the "feeling" for the robot.

In particular, the interface allows user to understand the behavior of an existing robot, and to investigate the performance of a newly designed structures without the need and the cost associated with the hardware implementation.

SimMechanics offers the possibility to visualize and animate the robot. The visualization tool can also be used to animate the motion of the system during simulation. The bodies of the robot can be represented as convex hulls (Fig. 8-9).

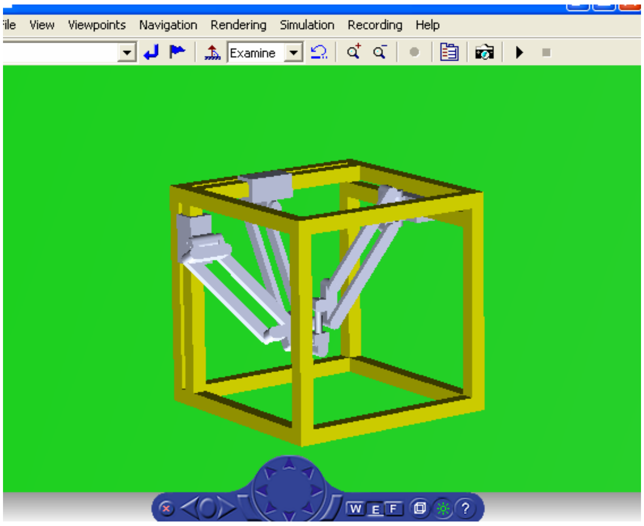

Fig. 8. Virtual Reality model of parallel robot de type Orthoglide.

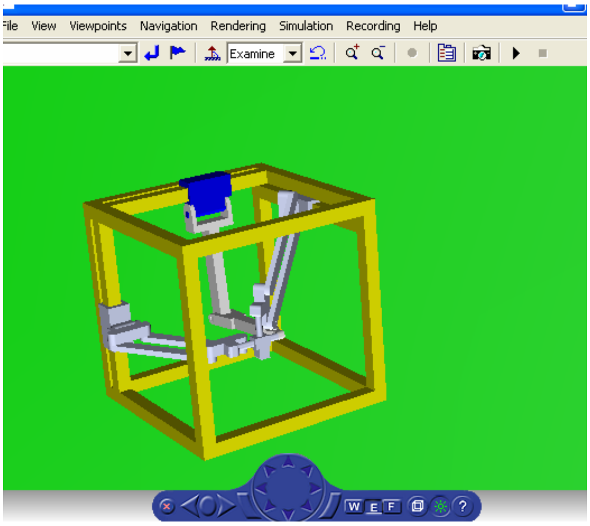

Fig. 9. Virtual Reality model of parallel robot de type 1PRRR+2PRPaR.

Dynamics Modeling of the Parallel Robots

The dynamic modelling has made using the Lagrange multipliers method in the rigid link hypothesis, where the gravity acceleration is considering on Z axe in negative sense.

Lagrange equation with multipliers is described by:

$$\sum_{i=1}^{k}\lambda_i\frac{\partial\Gamma_i}{\partial q_j}=\frac{d}{dt}\left(\frac{\partial L}{\partial q_j}\right)-\frac{\partial L}{\partial q_j}-\hat{Q}_j\ , \tag{7}$$

where: λ_i - Lagrange multipliers; L - Lagrangian of the system; q_j - displacement of the actuators; $\hat{Q}_j$ - generalizing external force.

The Lagrangian of the system is done:

$$L=\sum_{i=1}^{n}E_{ci}+\sum_{i=1}^{n}E_{pi}\ , \tag{8}$$

where: E_{ci} - kinetic energy; E_{pi} - potential energy;

The next step is to identify equations set for introduction of the multipliers (Γ_i):

- for Orthoglide parallel robot:

$$\Gamma=\begin{bmatrix}\Gamma_1\\\Gamma_2\\\Gamma_3\\\Gamma_4\\\Gamma_5\\\Gamma_6\end{bmatrix}=\begin{bmatrix}\sin(\varphi_{3a})\cdot l_{4a}-q_2-\cos(\varphi_{2b})\cdot\cos(\varphi_{3b})\cdot l_{4b}+l_{7b}\\ -\frac{1}{2}\cdot\sin(\varphi_{2a}+\varphi_{3a})\cdot l_{4a}-\frac{1}{2}\cdot\sin(\varphi_{2a}-\varphi_{3a})\cdot l_{4a}-q_3-\cos(\varphi_{2c})\cdot\cos(\varphi_{3c})\cdot l_{4c}+l_{7c}\\ -\sin(\varphi_{2b})\cdot\cos(\varphi_{3b})\cdot l_{4b}-q_1-\frac{1}{2}\cdot\cos(\varphi_{2a}+\varphi_{3a})\cdot l_{4a}-\frac{1}{2}\cdot\cos(\varphi_{2a}-\varphi_{3a})\cdot l_{4a}+l_{7a}\\ \sin(\varphi_{3b})\cdot l_{4b}-q_3-\cos(\varphi_{2c})\cdot\cos(\varphi_{3c})\cdot l_{4c}+l_{7c}\\ \sin(\varphi_{3c})\cdot l_{4c}-q_1-\frac{1}{2}\cdot\cos(\varphi_{2a}+\varphi_{3a})\cdot l_{4a}-\frac{1}{2}\cdot\cos(\varphi_{2a}-\varphi_{3a})\cdot l_{4a}+l_{7a}\\ -\sin(\varphi_{2c})\cdot\cos(\varphi_{3c})\cdot l_{4c}-q_2-\cos(\varphi_{2b})\cdot\cos(\varphi_{3b})\cdot l_{4b}+l_{7b}\end{bmatrix}\tag{9}$$

- for 1PRRR+2PRPaR parallel robot:

$$\Gamma=\begin{bmatrix}\Gamma_1\\\Gamma_2\\\Gamma_3\\\Gamma_4\\\Gamma_5\\\Gamma_6\end{bmatrix}=\begin{bmatrix}\cos(\varphi_{2a})\cdot l_{3a}+l_{4a}\cdot\cos(\varphi_{2a}+\varphi_{3a})-q_2-\cos(\varphi_{2b})\cdot\cos(\varphi_{3b})\cdot l_{4b}+l_{7b}\\ \sin(\varphi_{2a})\cdot l_{3a}+l_{4a}\cdot\sin(\varphi_{2a}+\varphi_{3a})-q_3-\cos(\varphi_{2c})\cdot\cos(\varphi_{3c})\cdot l_{4c}+l_{7c}\\ -\sin(\varphi_{2b})\cdot\cos(\varphi_{3b})\cdot l_{3b}-q_1+l_{7a}\\ \sin(\varphi_{3b})\cdot l_{4b}-q_3-\cos(\varphi_{2c})\cdot\cos(\varphi_{3c})\cdot l_{4c}+l_{7c}\\ \sin(\varphi_{3c})\cdot l_{4c}-q_1+l_{7a}\\ -\sin(\varphi_{2c})\cdot\cos(\varphi_{3c})\cdot l_{4c}-q_2-\cos(\varphi_{2b})\cdot\cos(\varphi_{3b})\cdot l_{4b}+l_{7b}\end{bmatrix}\tag{10}$$

Finally, the analytical expression of the driver forces Fq_1, Fq_2 and Fq_3 (see Fig. 1 and 2) are obtained.

Finally, the analytical expression of the driver forces Fq1, Fq2 and Fq3 (see Fig. 1 and 2) are obtained. Even the analytical dynamic model is complex; the method used can be applied to derive the dynamical model of any parallel robots with 3 DOF and can be extending for 4 and more DOF parallel robots.

Simulation Results

For testing the obtained models, it has been chosen a trajectory between two points in Cartesian space after polynomial low of fifth degree order. Regarding the displacements, it can be seen that for the same desired displacement of the end-effector for both parallel robots, the required displacements in motor joints in case of the coupled motions (for Orthoglide and for q_2 and q_3 from 1PRRR+2PRPaR) are bigger.

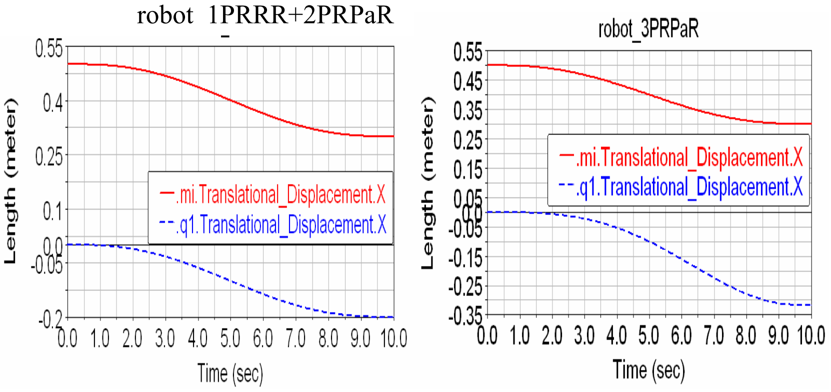

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Fig. 10. Displacement after X axis for the end-effectors (red-continues line) and for q1 (blue – dashed line) of parallel robot de type 1PRRR+2PRPaR (a) and Orthoglide (b).

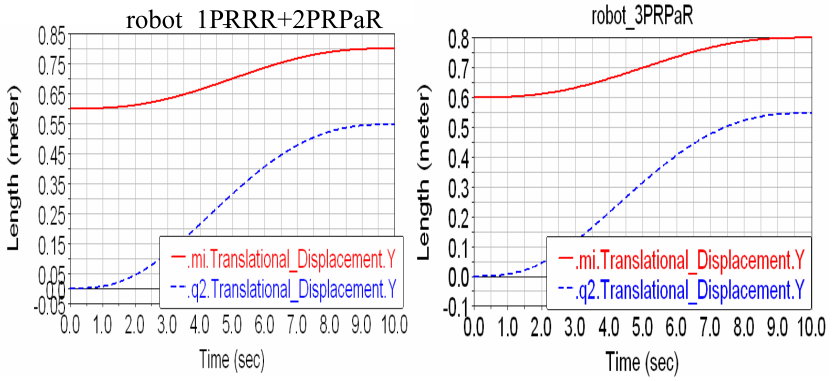

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Fig. 11. Displacement after Y axis for the end-effectors (red-continues line) and for q1 (blue – dashed line) of parallel robot de type 1PRRR+2PRPaR (a) and Orthoglide (b).

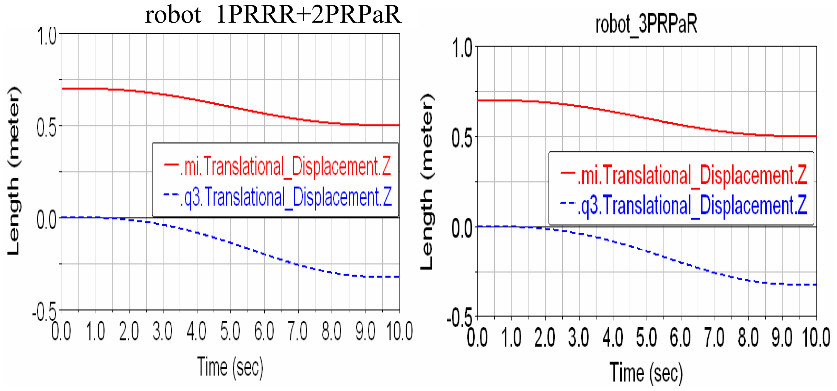

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Fig. 12. Displacement after Z axis for the end-effectors (red-continues line) and for q1 (blue – dashed line) of parallel robot de type 1PRRR+2PRPaR (a) and Orthoglide (b).

From the kinematic point of view it was noticed first a motion decoupling in the case of the parallel robot with 1 decoupled motions and on the other hand the influence of the motion coupling on the kinematic behavior. In the followings it will be studied also the influence of these coupling and decoupling on the developed forces in the motor joints.

It has to be mentioned that for studying comparatively these motor forces a total mass equivalent has been taken into consideration for both parallel robots. It can be seen that in for the parallel robot with one uncoupled motion (or two coupled motion) the forces developed in the translational joints is sensitive bigger comparative with the parallel robot with 3 coupled motions.

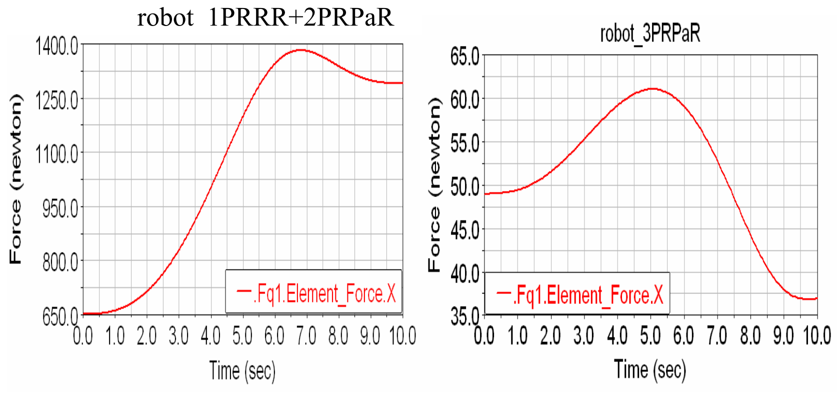

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Fig. 13. Driver forces Fq1 of parallel robot de type 1PRRR+2PRPaR (a) and Orthoglide (b).

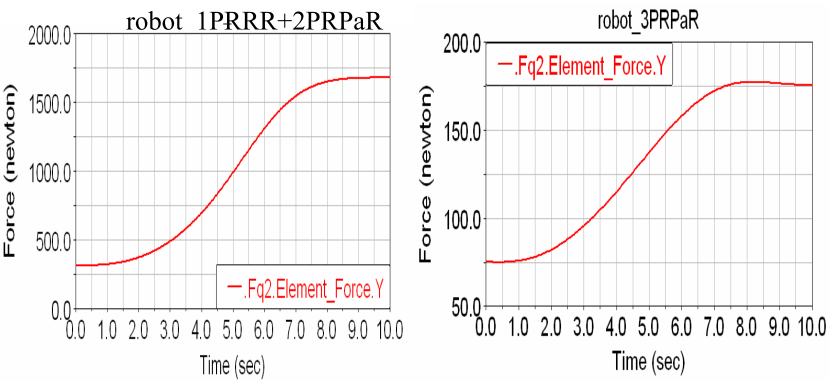

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Fig. 14. Driver forces Fq2 of parallel robot de type 1PRRR+2PRPaR (a) and Orthoglide (b).

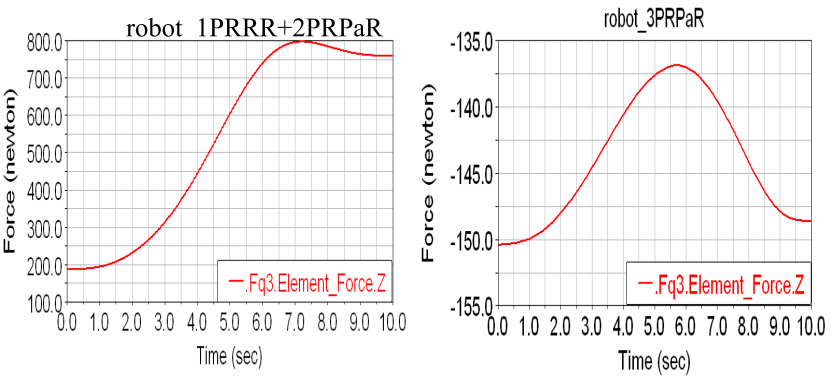

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Fig. 15. Driver forces Fq3 of parallel robot de type 1PRRR+2PRPaR (a) and Orthoglide (b).

Conclusions

The paper presents a novel Virtual Reality Interface for the 3 DOF medical parallel robots control. An evaluation model from the Matlab/SimMechanics environment was used for the simulation. An interactive tool for dynamics system modeling and analysis was presented and exemplified on the control in Virtual Reality environment of these medical parallel robots.

A dynamic modeling of maximally regular parallel robots [2], with application to the coupled topology has been presented in this paper. A closed form dynamic model has been set up in the rigid link hypothesis. The numerical simulations on a given trajectory emphasize the influence on the driving forces. With obtained analytical dynamic model for both parallel robots a control model can be add to complete the developed study.

The main advantages of this parallel manipulator are that all of the actuators can be attached directly to the base, that closed-form solutions are available for the forward and inverse kinematics, and that the moving platform maintains the same orientation throughout the entire workspace. By means of SimMechanics, the robotic system in represented as a block of functional diagrams. Besides, such software packages allow visualizing the motion of mechanical system in 3D virtual space. Especially non-experts will benefit from the proposed visualization tools, as they facilitate the modeling and the interpretation of results. The research presented will lay a good foundation for the development of medical parallel robots.

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