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Design sensitivity analysis in the kinematics of the 4SS-axle guiding mechanism in Chebyshev configuration

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Abstract. This work deals with the design sensitivity analysis in the kinematics of the vehicle rear axle guided by four points - on four spheres (so called 4SS) in Chebyshev configuration. The study actually continues the research presented in a previous published article, which addressed the effect of variations in geometric parameters on the kinematic behavior of the 4SS guiding mechanism with Panhard bar (that is, one of the 4 bars of the mechanism was arranged transversely, while now all the guide bars of the rear axle are arranged longitudinally). The aim of the research is to identify the geometric parameters with significant influence on the kinematics of the guiding mechanism, in order to simplify the subsequent effective optimization process.

1. Introduction

In relative movement to the car body (chassis), the rear axles of motor vehicles are guided by spatial linkages, which are constituted by driving three, four or five points of the axle on conveniently chosen surfaces and/or curves (spheres, arcs, connecting rod curves). The connections of the guide bars to axle and car body are made by compliant couplings (bushings), which support linear and angular deformations in all directions. Given that the linear deformations are very small (however, insignificant compared to angular deformities), they are frequently neglected, the bushings being modelled by spherical joints, thus reducing the complexity of the theoretical models [1-4].

A comprehensive systematization of the axle guiding mechanisms based on the modelling of bushings by spherical joints was presented in [1], being revealed two basic categories of mechanisms depending on the number of degrees of mobility (DOM), mono-mobile (DOM=1) and bi-mobile (DOM=2) mechanisms. However, it should be mentioned that the simplifying hypothesis bushings \leftrightarrow spherical joints proved to be valid only in the case of bi-mobile mechanisms, for the mono-mobile mechanisms being found that even very small linear deformations in bushings still influence the behaviour of the system (the space movement of the axle relative to the car body) [5].

From the category of bi-mobile mechanisms, one of the most frequently used variants is the guiding mechanism by four points - on four spheres (coded 4SS). Depending on the arrangement mode of the guide bars (in longitudinal-vertical or transversal-vertical plane), there are two basic configuration of 4SS mechanism, as shown in Figure 1. At the mechanism in Figure 1.a, three of the guide bars are in-parallel arranged in a longitudinal-vertical plane (thus taking over the longitudinal forces from the wheel-track contact), while the fourth bar (called Panhard bar) is transversally disposed in order to take over the transversal forces, while in the case of the mechanism in Figure 1.b all guide bars are arranged longitudinally (so called Chebyshev mechanism), but in a non-parallel configuration so that they are able to take over both longitudinal and transverse forces.



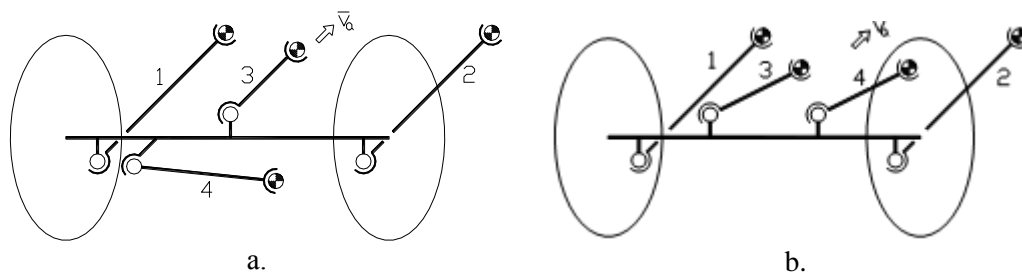


Figure 1. Basic configurations of 4SS guidance mechanisms.

The optimization of the guidance mechanisms is a challenge and a constant concern for research in the field of automotive engineering, given the crucial role that these mechanisms play on the behavior of the suspension system (mainly on the comfort and stability performances of the vehicle). The high complexity of kinematic and dynamic optimization processes is given in particular by the large number of geometric parameters that define these mechanisms (and which should be considered as independent design variables in an optimal design problem). This is why it would be useful to separate the geometric parameters into primary and secondary, so that in the effective optimization only the main parameters, with a significant influence on the specific kinematic or dynamic design objectives (i.e. objective functions) to be considered.

This work deals with the design sensitivity analysis in the kinematics of the 4SS guiding mechanism in Chebyshev configuration, the study actually continuing the research presented in a previous article by the author [6] in which the effect of variations in geometric parameters on the kinematic behavior of the 4SS guiding mechanism with Panhard bar (which is schematically shown in Figure 1.a) was approached, thus completing the problem of identifying the influence of geometric parameters on the kinematic behavior of 4SS type guidance mechanisms. The kinematic study is performed by the same method as that used in [6] (so called "characteristic points's method"), which is part of a more complex algorithm for determining the static balance position of the rear axle guidance mechanisms. The method was transposed on computer using the C++ programming environment.

2. Geometric and kinematic parameters

To define the geometric model of the guidance mechanism under study, two cartesian coordinate systems (reference frames) were defined/used, as shown in Figure 2, namely the global coordinate system OXYZ (attached to the car body, which is fixed part in kinematics), and the local coordinate system $PX_pY_pZ_p$ (attached to the rear axle). The global system is positioned in the middle of the axis of the front wheel centers, the X axis being the longitudinal axis of the vehicle (positive towards the rear of the vehicle), Y - the transverse axis, Z - the vertical axis. The local/axle coordinate system is placed in the middle of the rear axle, in the initial position (vehicle in rest) the local axes being oriented parallel to the global axes.

The geometrical parameters that define the guidance linkage are similar (as signification) to those used in [6], as follows: the global coordinates of the joint locations on car body (in OXYZ) - X_{M01} , X_{M02} , X_{M03} , X_{M04} ; the local coordinates of the joint locations on axle (in $PX_pY_pZ_p$) - $X_{M1(P)}$, $X_{M2(P)}$, $X_{M3(P)}$, $X_{M4(P)}$; the static (initial) position of the axle centre (in OXYZ) - X_p^0 , Z_p^0 . With the global coordinates of the points pair $M_{0i} - M_i$ ($i=1 \dots 4$), the lengths of the guide bars can be defined, $l_i = |M_{0i}M_i|$.

By considering the symmetry of the guidance mechanism relative to the longitudinal axis X, there are the following relationships between the global and local coordinates: $X_{M01} = X_{M02}$, $|Y_{M01}| = Y_{M02}$, $Z_{M01} = Z_{M02}$, $X_{M03} = X_{M04}$, $|Y_{M03}| = Y_{M04}$, $Z_{M03} = Z_{M04}$, $X_{M1(P)} = X_{M2(P)}$, $|Y_{M1(P)}| = Y_{M2(P)}$, $Z_{M1(P)} = Z_{M2(P)}$, $X_{M3(P)} = X_{M4(P)}$, $|Y_{M3(P)}| = Y_{M4(P)}$, $Z_{M3(P)} = Z_{M4(P)}$. Under these terms, the 4SS guidance mechanism in Chebyshev configuration is completely defined by 16 geometrical parameters: $X_{M01/2}$, $Y_{M01/2}$, $Z_{M01/2}$, $X_{M03/4}$, $Y_{M03/4}$, $Z_{M03/4}$, $X_{M1/2(P)}$, $Y_{M1/2(P)}$, $Z_{M1/2(P)}$, $X_{M3/4(P)}$, $Y_{M3/4(P)}$, $Z_{M3/4(P)}$, $l_{1/2}$, $l_{3/4}$, X_p^0 , Z_p^0 .

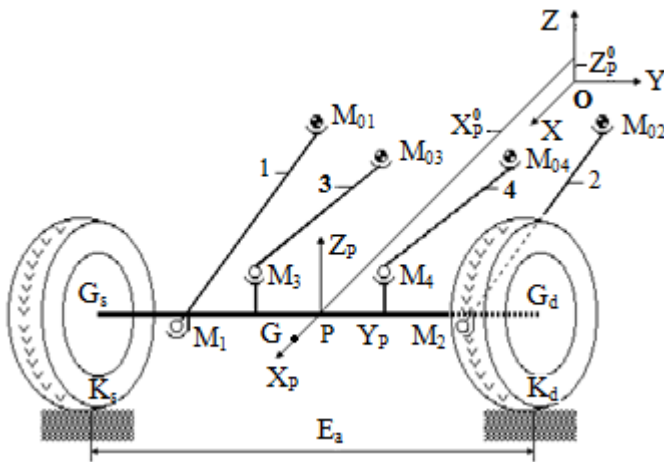


Figure 2. The geometric model of guidance mechanism.

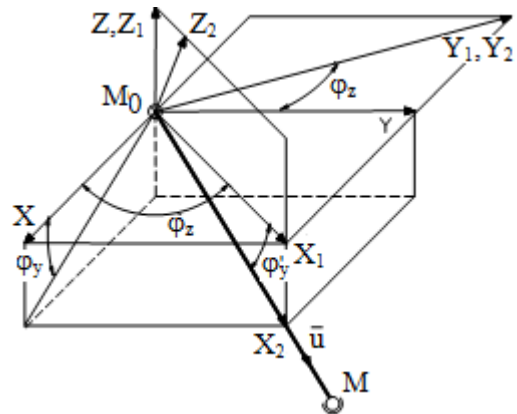


Figure 3. The positioning angles of the guide bars.

The spatial movement of the axle relative to the car body is defined by 6 kinematic parameters (linear and angular displacements), as follows: $\Delta X_p, \Delta Y_p, \Delta Z_p, \eta_{xy}, \eta_{xz}, \eta_{yz}$. Except for the vertical movement of the axle (ΔZ_p) and its rotation around the longitudinal axis (in the YZ plane, η_{yz}), which are the necessary movements of the bi-mobile mechanism (corresponding to the vertical movements of the wheel centres - Z_{G_s}, Z_{G_d} , which are independent kinematic parameters), the other 4 movements (namely $\Delta X_p, \Delta Y_p, \eta_{xy}$ and η_{xz}) are design objectives in the optimal design process, the kinematic optimization goal being to minimize these variations. The analytical forms of the kinematic functions used as design objectives have been defined in [1].

The local coordinates of the guidance points on axle ($X_{M_i(P)}, Y_{M_i(P)}, Z_{M_i(P)}, i=1..4$) are frequently established by constructive requirements. In the following, the influence of the other parameters on the kinematic behavior of the 4SS mechanism (in fact, on the unwanted linear and angular movements) will be evaluated.

The spatial disposing (arrangement) of the guide bars 1-4 is defined in accordance with the positioning angles shown in Figure 3, the global coordinates of joint locations on car body ($X_{M_{0i}}, Y_{M_{0i}}, Z_{M_{0i}}$) being expressed by the following equations:

$$X_{M_{0i}} = X_{M_i} - l_i \cos \varphi_{iz} \cdot \cos \varphi'_{iy}, Y_{M_{0i}} = Y_{M_i} - l_i \sin \varphi_{iz} \cdot \cos \varphi'_{iy}, Z_{M_{0i}} = Z_{M_i} + l_i \cdot \sin \varphi'_{iy}, \varphi'_{iy} = \arctan(\tan \varphi_{iy} \cdot \cos \varphi_{iz}), (1)$$

where the global coordinates of the interest points on axle ($X_{M_i}, Y_{M_i}, Z_{M_i}$) correspond to the known initial position of the guidance mechanism. Noting with k the ratio between the upper and lower bars' length ($k = l_3 / l_1$), there will result seven geometrical parameters whose influence on the kinematic design objectives ($\Delta X_p, \Delta Y_p, \eta_{xy}, \eta_{xz}$) will be analysed: $l_1, k, \varphi_{1y}, \varphi_{3y}, \varphi_{1z}, \varphi_{3z}$. The study is performed starting from a basic variant of the mechanism, in which each geometric parameter is modified individually, keeping the other parameters at the values corresponding to the basic variant.

3. Results and conclusions

Following the design sensitivity analysis for the axle guidance mechanism in study, the variation diagrams of the kinematic functions were obtained, as shown in Figures 4-9. The results correspond to the case in which the vertical displacements of the left and right wheels are equal and in the same direction, the variation range of the independent kinematic parameters being $\Delta Z_{G_{s,d}} \in [-80, 80]$ mm in relation with the initial state of the mechanism (when the car is at rest / stationary). In the design sensitivity analysis process, for each of the six geometric parameters selected according to those mentioned in the previous section of the paper, four representative values were considered, as follows: $l_1 = \{400, 450, 500, 550\}$ mm, $k = \{0.3, 0.5, 0.7, 0.9\}$, $\varphi_{1y} = \{-6, -2, 2, 6\}^\circ$, $\varphi_{3y} = \{-6, -2, 2, 6\}^\circ$, $\varphi_{1z} = \{-20, -10, 0, 10\}^\circ$, $\varphi_{3z} = \{-45, -15, 15, 45\}^\circ$.

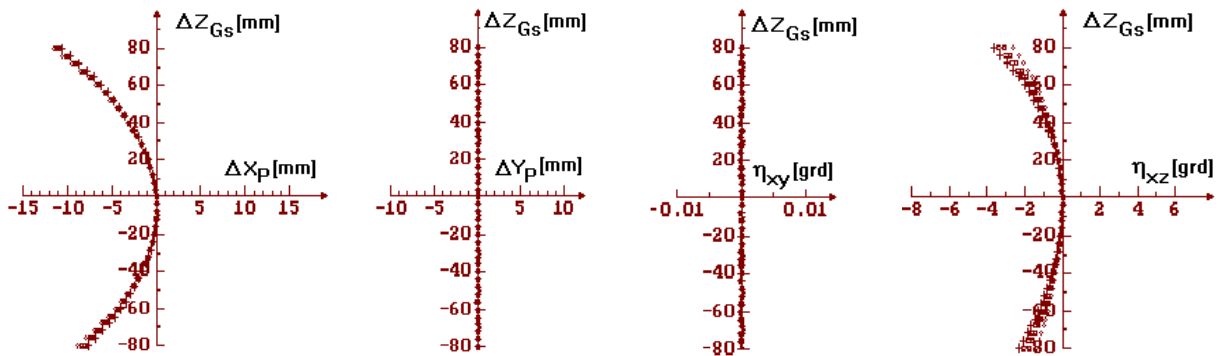


Figure 4. Design sensitivity analysis for the parameter l_1 .

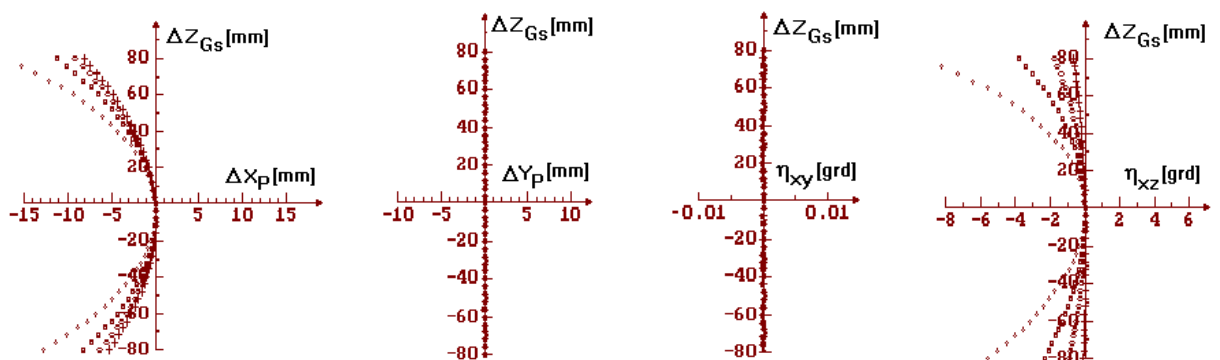


Figure 5. Design sensitivity analysis for the parameter k .

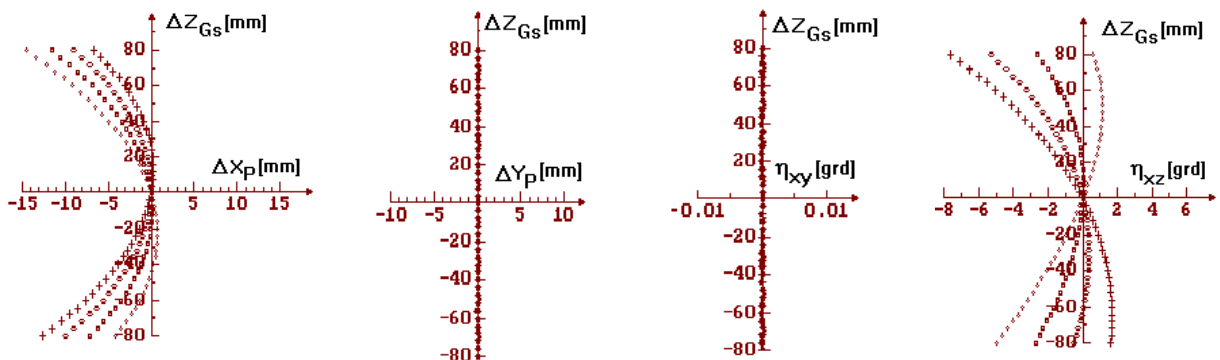


Figure 6. Design sensitivity analysis for the parameter ϕ_{1y} .

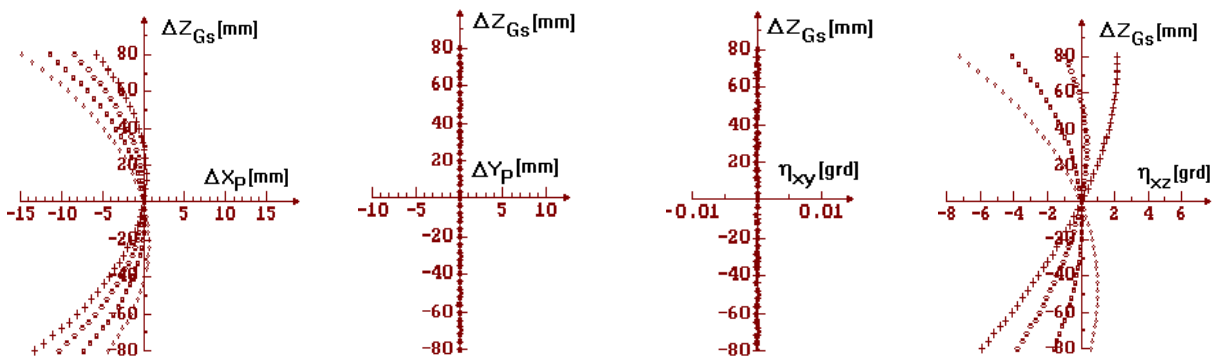


Figure 7. Design sensitivity analysis for the parameter ϕ_{3y} .

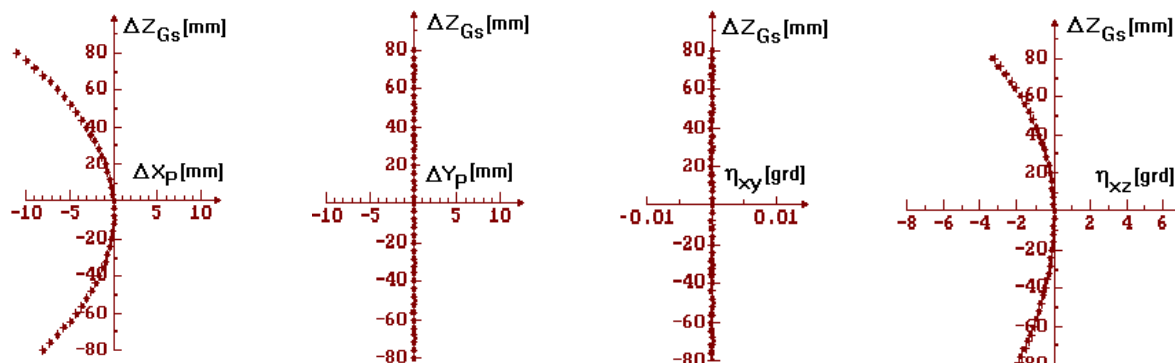


Figure 8. Design sensitivity analysis for the parameter φ_{1z} .

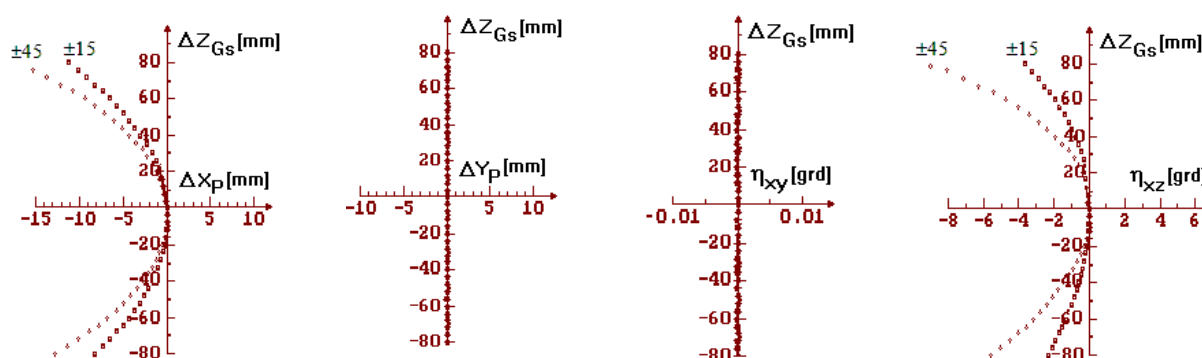


Figure 9. Design sensitivity analysis for the parameter φ_{3z} .

From the analysis of the results presented in the previous figures, the parameters with significant influence on the considered kinematic functions can be identified. Thus, the displacement of the axle along the longitudinal axis and its rotation in the longitudinal - vertical plane have a significant sensitivity to the change of most geometric parameters, $\Delta X_P \cong F(k, \varphi_{1y}, \varphi_{3y}, \varphi_{3z})$, $\eta_{xz} \cong F(l_1, k, \varphi_{1y}, \varphi_{3y}, \varphi_{3z})$, while the other two kinematic functions, ΔY_P and η_{xy} , are influenced to a very small extent (practically, insignificantly) by the rational changes of the geometric parameters. Consequently, in the kinematic optimization of the guidance mechanism, the geometric parameters that influence to a small extent the kinematic functions can be neglected, being retained only the main design variables ($k, \varphi_{1y}, \varphi_{3y}, \varphi_{3z}$) which translates into simplifying the effective optimization.

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