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Modeling the Economic Performance of Industrial Systems Using Mathematical Programming

FECHETE Flavia [a] and NEDELCU Anișor [b]

Transylvania University of Brasov, Manufacturing Engineering Department,
B-dul Eroilor, nr. 29, 500036, Brașov, Romania.

[a] fechete_flavia@yahoo.com, [b] a.nedelcu@unitbv.ro

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Abstract. Mathematical programming models and especially their subclass - linear programming models - plays an extremely important role, both in theory and in economic practice. Linear programming, through its results, brought a considerable contribution to improving management methods in economics and it has boosted theoretical research in modeling complex economic systems, study and interpretation of laws and economic processes. Developing and designing a model for achieving the economic performance of the industrial system allows managers making optimal decision and ensures the improvement of their activity. This paper aim is to determine an optimal manufacturing program for an industrial system, so that, by its implementation, to achieve economic performance. The manufacturing program conducted using a computer software will allow this entity to optimize their management decision process by providing information related to physical production that must be executed on each of their the products, or about the unused or overloaded capacity, in order to maximize their profits.

Introduction

The decision is the focus of management activity since it was in all its functions, and more integration within the company environment depends on the quality of the decision. At the same time, the quality of decision has an important influence on cost reduction, efficient use of funds, profit growth, etc.

Modeling and optimization techniques are used not only in the traditional supply chain management, but also in other functional areas of organizations as finance, accounting, marketing.

A large class of optimization models in economics is solved by linear programming. The first general method for solving the problem of linear programming (simplex algorithm) is due to G. Dantzig (1951) and determines solutions if they exist, or can prove the non-existence of solutions, their degeneration etc. A second general method for solving linear programming problem (dual Simplex algorithm) is due to EC Lemke (1953). Developing the two methods led to further research in this area and the results became chapters related to integer programming problems (algorithms R. Gomory, 1958, 1963), transport problems, problems of allocation (distribution). Subsequently, methods of solving diversified closely with the development of computers.[1]

The main objective of the economic processes modeling is to optimize the decisional process, as a determining factor in the transition from experience, intuition complex, a scientific approach based on information-reasoning.

The present work aims to help the company to improve decision management system, to create a decision model that will contribute to improvement. The major objective of this paper is to create a model for the economic performance of an industrial system, a model that will help the managers to make the best decisions on production optimization.

In this regard, it was made a brief introduction to the theory of mathematical programming, following with a case study in order to develop the linear programming used to determine an optimal production program. The case study was conducted on a system that produces and sells industrial fasteners and fastening systems. The research was focused on four products manufactured by the organization, which was intended to determine the optimal production program that maximizes profits. Thus, we determined the objective function and its constraints and using the linear programming module of the software QM, we were able to determine the amount of each product that must be manufactured in order to maximize profit.

Also, we made an economic analysis of the results obtained in QM program in terms of profits variation for each product, as well as spare production capacity.

Method

The decision process involves evaluating several decisional alternatives based on economic indicators, in order to choose the optimal variant. For choosing the optimal decision is necessary to hierarchy available variants, depending on the desired criteria. One of the most used methods in this regard it is economic modeling by linear programming.

Linear programming problem is within the general mathematical programming models and it is characterized by the fact that both the objective function and the constraints are expressed mathematically by linear functions. [2]

The linear programming problem contains restrictions, as well as a criteria of "performance" to assess the effectiveness of each activity. Depending on the purpose, we can choose as a criterion of efficiency an indicator measuring the effort, one that measures the result or an indicator expressed as a ratio of result and effort (or effort and result).

Maximum efficiency means minimizing the effort and maximizing the outcome and the optimal concept is defined in this case as a program that minimizes or maximizes an objective function and simultaneously satisfies all technical and economic restrictions. [3]

The general form of linear programming problem is [1] :

$$\text{Max (Min) } f(x) = c_1 \cdot x_1 + c_2 \cdot x_2 + \ldots + c_n \cdot x_n. \quad (1)$$

With the following constraints:

$$\begin{aligned} & a_{11} \cdot x_1 + a_{12} \cdot x_2 + \ldots + a_{1n} \cdot x_n \leq b_1 \\ & a_{21} \cdot x_1 + a_{22} \cdot x_2 + \ldots + a_{2n} \cdot x_n \leq b_2 \\ & \ldots\ldots \\ & a_{m1} \cdot x_1 + a_{m2} \cdot x_2 + \ldots + a_{mn} \cdot x_n \leq b_m \\ & x_1 \geq 0, x_2 \geq 0, \ldots, x_n \geq 0. \end{aligned} \quad (2)$$

In practice, many applications in economics are solved using linear programming. One of the situations in which linear programming is successfully applied is for production scheduling. Determining the optimal production program is one of the major concerns of managers because it is closely related to the economic performance of the organization, deciding the success of the organization.[4]

There is presented below a first case of economic performance modeling of the organization by using linear programming. There is considered a production process in which are manufacturing n products / activities. For the manufacturing process there are available m resources (raw materials, labor, production capacity, financial resources), limited up. There are known:

- quantities of resources required to produce one unit of product /activitity j.
- production costs and selling prices per unit of product / activitity j.

Annotations:

b_i, i = 1,2,..., m – the known level of resources.

a_{ij}, i = 1,2,..., m, j =1,2,...,n - the amount of the i resource used to produce a unit of product / activity j.

c_j, j = 1,2,...,n - selling price of a unit of product / activity j.

d_j, , j =1,2,...,n - production cost of one unit of product / activity j.

x_j , j = 1,2,...,n – the unknown level of the product / activity j.

It is required to determine the levels of n items / activities that lead to maximum benefit.
The problem leads to the construction of the next model which is solved by linear programming method.
The objective function [5]:

$$\max \sum_{j=1}^{n}(c_j - d_j)\cdot x_j. \quad (3)$$

Constraints [5]:

$$\sum_{j=1}^{n}(a_{ij} \cdot x_j) \leq b_i,\ i = 1,2,\ldots,m\ \ x_j \geq 0. \quad (4)$$

Results

In order to proper understand the concept, it will be presented a case study on a industrial system that produces and sells assembling and fastening systems, case study illustrating the application of mathematical models of linear programming.

The industrial system is manufacturing the product screw, that is made in four versions: cylindrical screws, square head screws, fly-headed screw and eyebolt.
The manufacturing process consists of the implementation of the following successive operations: prepare the drawing of castings (O1), preparation of forming mixture (O2), top casting (O3), assembly the forms (O4), opening and dragging out the castings from the mould (O5), final control of castings (O6).
Execution times per unit of product (in hours) are shown in Table 1.

Table 1. Execution time/ product (hours)

Product	O_1	O_2	O_3	O_4	O_5	O_6
Cylindrical screws	0.2	0.4	0.7	1	0.3	0.2
Square head screws	0.2	0.5	0.7	0.5	0.8	0.3
Fly-headed screw	0.3	0.5	0.5	0.7	0.2	0.4
Eyebolt	0.4	0.5	0.9	0.2	0.7	0.5

The unit profit obtained is 13 monetary units (m.u.) for cylindrical screws, 19 m.u. for square head screw, 14 m.u. for fly-headed screw and 20 m.u. for the eyebolt.
Estimated time available for each operation are shown in Table 2.

Table 2. Machine available time (hours)

Operation	O_1	O_2	O_3	O_4	O_5	O_6
Available time	400	520	650	750	400	350

The case study aims to determine the optimal production program, i.e. the number of products in the four variants to be executed in order to maximize the total profit obtained.
Annotations: x_1 – number of the cylindrical screws
x_2 – number of the square head screws
x_3 – number of the fly-headed screw
x_4 – number of the eyebolt
The obtained model will be:

$$\mathrm{Max}(13\cdot x_1+19\cdot x_2+14\cdot x_3+20\cdot x_4). \quad (5)$$

Constraints :

$0.2\cdot x_1 + 0.2\cdot x_2 + 0.3\cdot x_3 + 0.4\cdot x_4 \leq 400.$
$0.4\cdot x_1 + 0.5\cdot x_2 + 0.5\cdot x_3 + 0.5\cdot x_4 \leq 520.$

$$0.7\cdot x_1 + 0.7\cdot x_2 + 0.5\cdot x_3 + 0.9\cdot x_4 \le 650.$$
$$1\cdot x_1 + 0.5\cdot x_2 + 0.7\cdot x_3 + 0.2\cdot x_4 \le 750.$$
$$0.3\cdot x_1 + 0.8\cdot x_2 + 0.2\cdot x_3 + 0.7\cdot x_4 \le 400.$$
$$0.2\cdot x_1 + 0.3\cdot x_2 + 0.4\cdot x_3 + 0.5\cdot x_4 \le 350. \quad (6)$$

To determine the optimal production program we will use the Linear Programming module of the QM (Quantitative Management) software.

Linear Programming Results

(untitled) Solut

	X1	X2	X3	X4		RHS	Dual
Maximize	13,	19,	14,	20,			
Constraint 1	0,2	0,2	0,3	0,4	<=	400,	0,
Constraint 2	0,4	0,5	0,5	0,5	<=	520,	0,
Constraint 3	0,7	0,7	0,5	0,9	<=	650,	0,
Constraint 4	1,	0,5	0,7	0,2	<=	750,	5,1462
Constraint 5	0,3	0,8	0,2	0,7	<=	400,	13,2749
Constraint 6	0,2	0,3	0,4	0,5	<=	350,	19,3567
Solution->	277,7778	222,2222	500,	55,5556		$15.944,44	

Fig. 1. Determining the optimal production program in QM using Linear Programming module

Fig. 1 shows the obtained solution - optimal program involves the development of 277,778 cylindrical screws, of 222.22 square head screws, 500 fly-headed screw and 55.5556 eyebolt. The corresponding value of the objective function will be 15 944.44 m.u.

Because the products manufactured are indivisible, the optimal solution will be sought in the nonnegative integers $\mathbb{Z}_+$ set. Among the methods of finding the solution in positive integers we have [6]:

- Method "Branch and Bound".
- Method due to R. Gomory;

Both methods can use the simplex algorithm, to the initial model new restrictions are added, obtaining a number of problems, until it is determined the optimal solution in $\mathbb{Z}_+$.

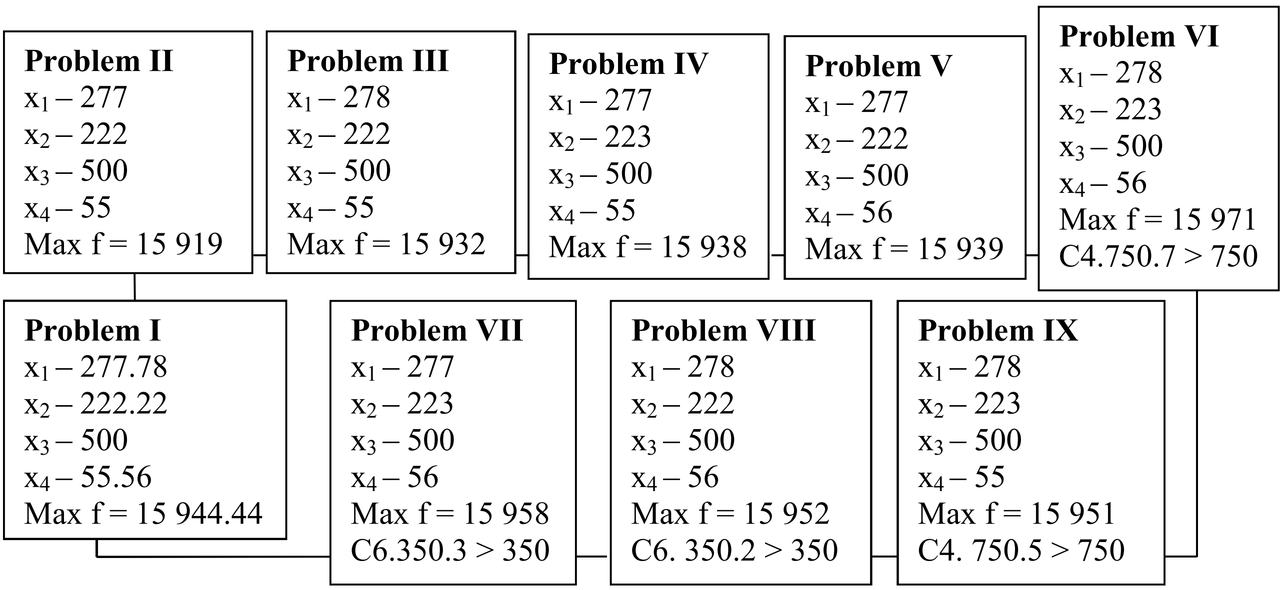

Fig. 2. The optimal solution of the problem in integers using Branch and Bound method

By solving the algorithm with Branch and Bound method, we identify the optimal solution in problem V (Fig. 2), which allows getting maximum profit by satisfying all constraints.

Using Linear Programming module of the software QM allows an economic analysis and interpretation of the solutions obtained. Fig. 3 shows the results of this analysis.

Ranging

(untitled) Solution					
Variable	Value	Reduced Cost	Original Val	Lower Bound	Upper Bound
X1	277,78	0	13	13	19,82
X2	222,22	0	19	14,3	19
X3	500	0	14	9,1	14
X4	55,56	0	20	20	23,87
Constraint	Dual Value	Slack/Surplus	Original Val	Lower Bound	Upper Bound
Constraint 1	0	127,78	400	272,22	Infinity
Constraint 2	0	20	520	500	Infinity
Constraint 3	0	0	650	627,78	738,89
Constraint 4	5,15	0	750	492,37	791,76
Constraint 5	13,27	0	400	321,24	442,7
Constraint 6	19,36	0	350	223,33	373,03

Fig. 3. Economic analysis and interpretation of the results obtained in QM

Reduced Costs – shows how much will need to modify the objective function coefficients, so that the final solution values of the variables to be positive.

Slack / Surplus - provide auxiliary variable values for each restriction. In this studied example, it gives important information for the manager, related to the spare production capacity. Thus, we can see that we have an excess of 127.78 hours for the operation 1, and for the operation 2 an additional of 20 hours.

Dual Value - gives information on the marginal value of resources in the optimal solution. The dual price associated with a restriction shows how much the optimal value of objective function improves when we increase by one unit the value of the right member of the restriction.

Thus, if we increase with one hour the time allotted for operation 5, from 400 hours to 401 hours it will result an increase in the objective function with 13.28 m.u., from 15 944.44 m.u. to 15 957.72 m.u. (Figure 4). Clearly, the operations 1, 2 and 3, the dual Value = 0, because to these operations we have found an additional unused hours.

Linear Programming Results

						(untitled) Solution	
	X1	X2	X3	X4		RHS	Dual
Maximize	13,	19,	14,	20,			
Constraint 1	0,2	0,2	0,3	0,4	<=	400,	0,
Constraint 2	0,4	0,5	0,5	0,5	<=	520,	0,
Constraint 3	0,7	0,7	0,5	0,9	<=	650,	0,
Constraint 4	1,	0,5	0,7	0,2	<=	750,	5,1462
Constraint 5	0,3	0,8	0,2	0,7	<=	401,	13,2749
Constraint 6	0,2	0,3	0,4	0,5	<=	350,	19,3567
Solution->	276,5351	225,0438	500,1315	54,2544		$15.957,72	

Fig. 4.The variation of the objective function after increasing by one unit the time of the operation 5

Objective Coefficient Ranges - for the coefficients change of the objective function between: (Lower Bound - Upper Bound), the optimal solution remains unchanged. From Figure 3, it follows that a change in the profit of the product – cylindrical screw – between 13 m.u. - 19.82 m.u. would not lead to changes in the optimal structure of production.

Instead, for a profit of 20 m.u. for the cylindrical screw, the manufacturing of the fly-headed screw is completely abandoned, and the industrial system will produce only cylindrical screw, square head screws and eyebolts (Fig. 5).

Finally, the Right Hand Side Ranges –dual price restrictions associated remains valid if the value of the resource (the right member of inequality) varies in the range (Lower Bound - Upper Bound). For example, in Fig. 4 we deduce that dual price of 5.1462 m.u. remains valid if the operation time varies in the range (492.37 - 791.7582) hours.

Linear Programming Results

(untitled) Solution

	X1	X2	X3	X4		RHS	Dual
Maximize	20,	19,	14,	20,			
Constraint 1	0,2	0,2	0,3	0,4	<=	400,	0,
Constraint 2	0,4	0,5	0,5	0,5	<=	520,	0,
Constraint 3	0,7	0,7	0,5	0,9	<=	650,	17,1852
Constraint 4	1,	0,5	0,7	0,2	<=	750,	6,5926
Constraint 5	0,3	0,8	0,2	0,7	<=	400,	4,5926
Constraint 6	0,2	0,3	0,4	0,5	<=	350,	0,
Solution->	637,037	196,2962	0,	74,0741		$17.951,85	

Fig. 5. The new manufacturing program at the profit variation for the cylindrical screw product

Thus if we go outside the upper and lower bound the shadow price shown is no longer valid. For increasing the value at 792 at operation 4, we wiil see that the dual price is zero and there is even an unused time. That may imply unwanted extra costs for the organization.

Conclusions

The main objective of business process modeling is to optimize decision-making process, as a determining factor in the transition from experience, intuition complex, to a scientific approach based on information-reasoning. Economic modeling process enables the manager making the best decision, giving it multiple ways to make an efficient agreement between human, material, financial with the main objectives of the organization.

This paper objective is to help managers to improve the economic performance of the organization by determining and implementing an optimal production program. Modeling a manufacturing program using the QM software led to obtaining precise quantities to be produced by the organization in order to maximize their profits, while allowing a rigorous economic analysis of variance profits for each product or manufacturing capabilities.

Future research will focus on multi-criteria modeling methods, namely the multidimensional linear programming that allows to use the objective function as a utility function that will combine both criteria of maximizing profits and minimizing costs.

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