ON THE PROBABLE MAXIMUM PRECIPITATION METHOD

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Abstract. Climate change is affecting the population's life and activity all over the world. Precipitation is one of the meteorological vectors essential for normal plant growth and agricultural activities. The *Probable Maximum Precipitation* (PMP) concept is based on the evaluation, using a deterministic approach, of the upper limit of the precipitation at a location, in a given period. In this context, the study aims at determining the *Probable Maximum Precipitation* (PMP) in Dobrogea, Romania, a region experiencing long drought periods. Since the Hershfield method, recommended by WMO, does not provide an appropriate PMP value for the study region, two alternatives are proposed. Comparisons are also provided.

Key words: precipitation; PMP; Hershfield method.

1. INTRODUCTION

Defined in the WMO documents as the highest rainfall quantity recorded for a particular period at a given site, the *Probable Maximum Precipitation* (PMP) has significant importance for predicting the apparition of extreme events [1]. More precisely, PMP is "the theoretically greatest depth of precipitation for a given duration that is physically possible over a given size storm area at a particular geographical location at a certain time of the year" [2].

The PMP concept relies on the hypotheses that an upper bound of the precipitation depth at a specified location and particular time of year exists and it can be determined [3] based on deterministic reasons [4].

Different approaches to estimate the PMP are known, the main ones being statistical and hydro-meteorological [5]. The first category is based on frequency analysis. The method developed by Hershfield [6, 7] belongs to the first group. Other authors [8, 9] consider that the methods for computing the PMP belong to one of the classes:

- models that take into account the storms;
- the actual storms maximization and transposition;

- generalized approaches;
- determination of empirical formula utilizing knowledge on the study area and maximum depth duration;
- utilization of empirical correlation between variables in certain valleys, in the case when such data are available;
- statistical analyses.

In the meteorological approach, PMP is determined from the individual observed storms' transposition and maximization. Adjustments for altitude, barriers for inflow moisture, and distance from the moisture source are included by the transposition. According to the Australian Bureau of Meteorology, generalized methods for estimating PMP utilize all the available records from an extended area. They also include adjustments for moisture availability and topographic effects on the rainfall depth. Generalized methods also provide the design of the PMP spatial and temporal patterns.

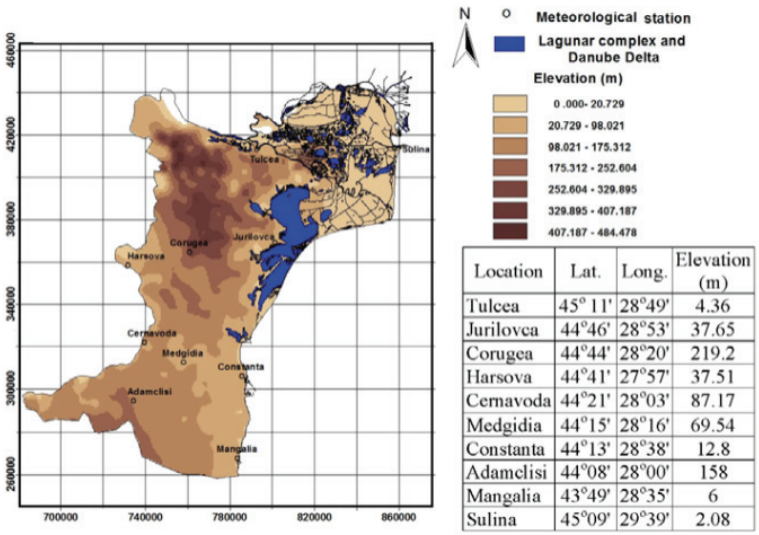
Koutsoyiannis [9] proposed a method to attach a return period to PMP, fitting a GEV distribution using the stations' record on which the original article of Hershfield [7] is based. He proved that Hershfield's PMP estimate might be obtained by his method.

In a similar study, Papalexiou and Koutsoyiannis [3] studied the PMP using the moisture maximization method. The exceedance risk for the PMP can be determined by the multifractal method as well [9]. Chavan and Srinivas [10] compared the results of the storm model and Hershfield approach for catchments in India, whereas Casas *et al*. [11] proposed the adjustment of the methodology when outliers exist in some time series. Singh *et al*. [12] and Ben Alaya *et al*. [13] evaluated the uncertainty in the PMP estimation for case studies from the USA and Canada.

In this context, our article discusses the PMP computation in the presence of the outliers, aiming at clarifying some aspects related to the particularities of the study region. Moreover, even if different studies treated the evolution of climate factors in Romania [14–18] none of them addressed the influence of outliers on the probable maximum precipitation evaluation, which is of interest for predicting the high floods.

2. MATERIALS AND METHODS

Data used in this study have been recorded at ten meteorological stations from the Dobrogea region, Romania (Fig. 1). We are dealing with the annual maximum daily precipitation series. The study sites, the length of the record, and the highest observed rainfall are presented in Table 1.



Location	Lat.	Long.	Elevation (m)
Tulcea	45° 11'	28°49'	4.36
Jurilovca	44°46'	28°53'	37.65
Corugea	44°44'	28°20'	219.2
Harsova	44°41'	27°57'	37.51
Cernavoda	44°21'	28°03'	87.17
Medgidia	44°15'	28°16'	69.54
Constanta	44°13'	28°38'	12.8
Adamclisi	44°08'	28°00'	158
Mangalia	43°49'	28°35'	6
Sulina	45°09'	29°39'	2.08

Fig. 1 – Study locations in the Dobrogea region.

Table 1

Study series and the highest annual daily record

Location	No. of years of records	Highest rainfall (mm)
Adamclisi	40	83.30
Cernavodă	41	73.6
Constanța	41	201
Corugea	36	114.8
Hârșova	39	96.7
Jurilovca	21	79.9
Mangalia	40	146
Medgidia	36	84.6
Sulina	40	84.9
Tulcea	36	134.5

Firstly, some statistical analyses for all data series have been performed. They include the Mann-Kendall test (to verify the hypothesis of a monotonic trend existence against the inexistence of such a trend) and the outliers detection (by the boxplot). The R software and Minitab 15 have been utilized.

The PMP (h_{max}) for a series is estimated from the equation [6, 7]:

$$h_{\max} = \overline{h_n} + k_{\max} s_{n'} \tag{1}$$

where:

– $\overline{h_n}$ and s_n are the mean and standard deviation at the same station, for which the corresponding series has n annual maximum rainfall values,

– k_m is the frequency factor related to the chosen station, calculated by the equation

$$h_m = \frac{h_1 - \overline{h_{n-1}}}{s_{n-1}}, \tag{2}$$

– h_1 is the highest value of data series,

– $\overline{h_{n-1}}$ and s_{n-1} are the mean and standard deviation of the series without h_1,

– k_{max} is the maximum of all frequency factors h_m in a given region.

In his study [6], Hershfield found k_m between 3 and 15, using data from more than 2600 stations worldwide and adopted for the PMP estimation the factor $k_{max} = 15$. Since the use of this value leads to the maximum precipitation overestimation, other scientists [10–13] employed different k values, estimated from adapted to the study series.

In our analysis, the following steps have been performed:

1. Determine k_{max} for the data series, and estimate the PMP by formula (1);
2. Determine k_{avg} as the average of all k_m computed by using (2), and estimate the PMP using (2) where k_{max} is replaced by k_{avg};
3. Remove the outliers, defined as those values in a time series outside the interval (mean – 2.5∗ standard deviation, mean + 2.5 ∗ standard deviation);
4. Compute k_m (denoted now $k_{m,o}$) with (2), for the series without outliers;
5. Determine the new PMP by (1), using the new k_{max} (denoted now $k_{max,o}$);
6. Use the procedure from step 3, but instead of $k_{max,o}$ use the average $k_{avg,o}$, defined as the average of the $k_{m,o}$.

Finally, the errors have been computed as the difference between the maximum recorded data and the estimated one. The results have been compared to find the best approach for the PMP evaluation in the study region.

3. RESULTS AND DISCUSSION

The Mann – Kendall test did not detect a significant trend for all annual maximum rainfall series but Cernavodă and Sulina series. Still, the PMP is overall applicable to all ten stations [5].

All histograms, but that of Medgidia are right-skewed. For example, Fig. 2 presents the histograms of the Medgidia and Hârşova series. The first one is symmetric, while the second one is right-skewed.

Figure 3 contains the boxplots of the study series. One can remark the outliers existence for Constanţa, Corugea, Hârşova, Mangalia, Medgidia, Sulina. They are 201 mm for Constanţa, 114.8 and 99.8 mm for Corugea, 96.7 mm for Hârşova, 79.9 mm for Jurilovca, 127 mm and 146 mm for Mangalia, 84.6 mm for Medgidia, 84.9 mm for Sulina, and 134.5 and 94.4 for Tulcea.

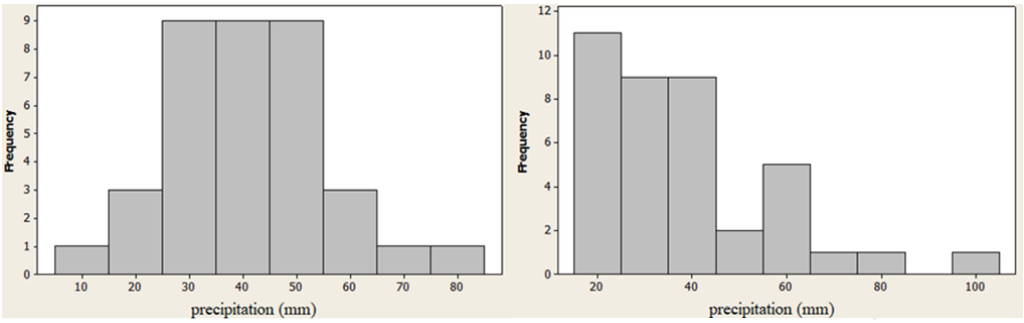


Fig. 2 – The histograms of Medgidia and Harsova series.

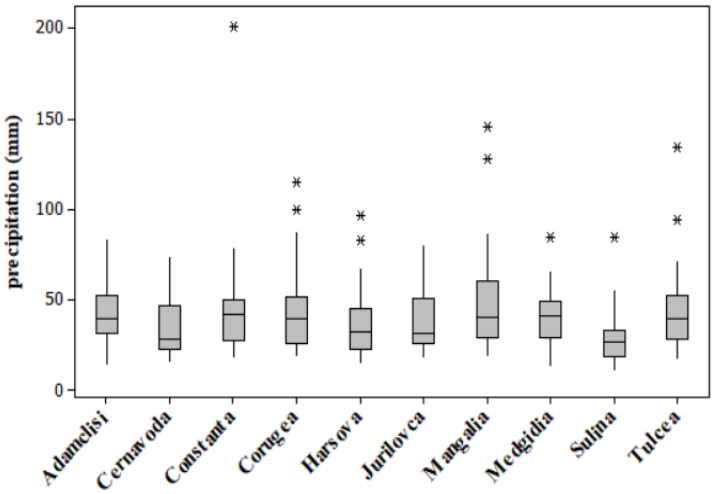


Fig. 3 – The boxplots of the study series.

The values of k_m, k_{max}, k_{avg}, $k_{m,o}$, $k_{max,o}$, and $k_{avg,o}$ are presented in Table 2. Analyzing this table, one remarks the high k_{max} value (column 3), and its significant decrease (column 2) after removing the outliers. A significant reduction is also noticed for k_{avg}, from 4.38 to 2.64. Comparing columns 2 and 5 of Table 2, one notices that after removing the outliers, k_m also diminished, most significantly for Constanța and Tulcea series.

Taking into account the values from Table 2, columns 3 and 4, Table 3 presents the maximum values recorded at each station, the estimated PMP using k_{max} and k_{avg}, and the corresponding errors, e_{max}, and e_{avg}, computed by the difference between PMP values and the recorded ones in each case.

Data from column 3 of Table 3 show a high overestimation of maximum precipitation when using k_{max}. The errors are not acceptable since they are generally higher than the records (Table 3, column 4).

Table 2

The values of k_m, k_{max}, k_{avg}, $k_{m,o}$, $k_{max,o}$, $k_{avg,o}$

Location	k_m	k_{max}	k_{avg}	$k_{m,o}$	$k_{max,o}$	$k_{avg,o}$
Adamclisi	2.86	10.32	4.38	2.16	3.96	2.64
Cernavodă	2.85			2.50		
Constanța	10.32			2.60		
Corugea	3.63			2.88		
Hârșova	3.75			2.50		
Jurilovca	2.82			2.46		
Mangalia	4.63			2.76		
Medgidia	3.45			1.99		
Sulina	3.93			3.96		
Tulcea	5.53			2.43		

Table 3

The maxima and PMP computed usingk_{max} andk_{avg}, and the corresponding errors

Location	Max	PMP with k_{max}	e_{max}	PMP with k_{avg}	e_{avg}
Adamclisi	83.30	206.72	123.42	112.49	29.19
Cernavodă	73.60	202.89	129.29	106.52	32.92
Constanţa	201.00	349.94	148.94	174.49	–26.51
Corugea	114.80	278.91	164.11	143.65	28.85
Hârșova	96.70	230.13	133.43	119.33	22.63
Jurilovca	79.90	208.66	128.76	110.32	30.42
Mangalia	146.00	322.69	176.69	164.74	18.74
Medgidia	84.60	195.78	111.18	106.65	22.05
Sulina	84.90	201.23	116.33	102.77	17.87
Tulcea	134.50	279.13	144.63	143.49	8.99

The PMPs computed using the average values of k_m are much better in terms of errors (last column from Table 3). The new errors are between 4.22 and 16.08 times smaller than those from column 3. Therefore, the use of k_{avg} is a better alternative to k_{max}. Still, when computing the PMP by utilizing k_{avg}, the maximum precipitation recorded at Constanţa was underestimated.

Table 4 contains the maxima after removing the outliers from each series, the estimated PMP using $k_{max,o}$ and $k_{avg,o}$, together with the corresponding errors $e_{max,o}$ and $e_{avg,o}$.

The results from Table 4 show superior performances of the maximum precipitation estimation after removing the outliers. When using $k_{max,o}$ in the computation (column 3), the errors ($e_{max,o}$ – column 4) are higher than when using the $k_{avg,o}$ (last column) for all but Sulina series. For this last series, the maximum precipitation is underestimated when utilizing $k_{avg,o}$.

Table 4

The maxima and PMP computed using $k_{max,o}$ and $k_{avg,o}$, and the corresponding errors

Location	Max	PMP with $k_{max,o}$	$e_{max,o}$	PMP with $k_{avg,o}$	$e_{avg,o}$
Adamclisi	70.80	99.20	28.40	9.94	9.14
Cernavodă	63.50	89.07	25.57	0.42	6.92
Constanţa	77.80	102.43	24.63	1.84	4.04
Corugea	86.80	111.54	24.74	7.74	0.94
Hârşova	71.20	97.20	26.00	6.89	5.69
Jurilovca	67.00	91.33	24.33	2.34	5.34
Mangalia	86.60	111.54	24.94	8.72	2.12
Medgidia	65.60	91.90	26.30	4.36	8.76
Sulina	84.30	85.20	0.90	6.26	–18.04
Tulcea	71.00	96.07	5.07	7.00	6.00

Comparing all the errors from Table 3 and Table 4, one may see that employing the average value of the individual k values provides the best approximations for PMP.

The PMP computation for the initial series using $k_{max,o}$ provides the results represented in Fig. 4. The PMP estimation for the initial maximum series, but using $k_{avg,o}$=3.96 (Figure 4) gave better results than when using k_{max} =10.32. In this case, the errors range between – 29.01 (for Constanţa) and +26.06 (for Cernavodă).

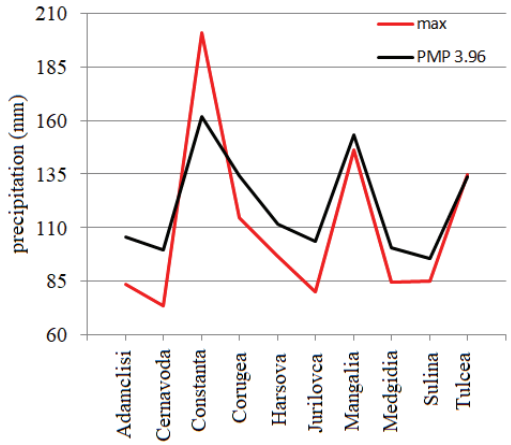


Fig. 4 – PMP for the initial maximum series using $k_{max,o}$ =3.96.

4. CONCLUSION

In this article, different alternatives for computing the Probable Maximum Precipitation for a set of ten series from the Dobrogea region have been analyzed. It was shown that the Hershfield method didn't provide good estimations of the PMP. Therefore, other alternatives have been proposed.

The best results have been obtained for series without outliers, employing the average value of individual k–s, not k_{max}, as prescribed in the initial method of Hershfield. Good approximations of PMP for the series containing outliers have been obtained utilizing the k_{avg} for the series without outliers. To confirm the validity of this approach for the study region, more investigations are necessary.

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