

# Static analysis of a multi-link suspension system used for the rear beam axles of vehicles

Catalin Alexandru<sup>1</sup>

<sup>1</sup> Product Design, Mechatronics and Environment Department, Transilvania University of Braşov, Romania

E-mail: calex@unitbv.ro

**Abstract.** The primary goal of the static analysis for the vehicle axle suspension systems consists in finding the equilibrium position of the mechanism in relation to the car body (which is considered the fixed reference part), under the action of the contact forces on wheels resulting from the overall dynamic balance of the vehicle. In the equilibrium position of the suspension, the reaction forces in the elastic elements of the suspension are to be determined, these being necessary in order to properly sizing these elements. In the present work, the previously stated problem is addressed by using an analytical method (and the corresponding computing program) that was detailed in an earlier work by the author. The study in this paper is focused on determining the reaction forces from the elastic elements of one of the most used types of axle suspension system, namely the one with a five-bar guidance mechanism.

**Keywords:** *beam, static analysis, elastic elements, guidance mechanism*

## 1. Introduction

The suspension system of the rear beam (rigid) axle of the vehicles is a dependent suspension design in which the left and right wheels are connected laterally by a single beam/shaft, thus existing only one suspension mechanism for both wheels (which implies reciprocal influence between the two parts of the suspension). Beam axle suspension is commonly used for commercial and off-road vehicles, unlike the passenger car where the independent wheel suspension design (at which each wheel has its own suspension mechanism) is commonly used.

The guidance mechanism in the beam axle suspension system contains a number of articulated bars/links (hence the name multi-link mechanisms), which are arranged in various configurations, both longitudinally and transversely. Reference [1] shows a detailed structural systematization of the rear beam axle guidance mechanisms, considering their number of degrees of mobility, as well as the number and arrangement of the guiding bars. The solution used to connect the guiding bars constituting the mechanism to the adjoining bodies/parts (i.e. axle and chassis) is the one with bushings (often called flexiblocks), which are compliant (deformable) connections that allow all six linear and angular degrees of freedom, but which are elastically restricted [2].

In addition to the multi-link mechanism that simultaneously guides the left and right wheels, the rear axle suspension system also includes specific elastic and damping elements, such as springs, anti-roll/stabilizer bar, bump stops (jounce bumpers) for limiting the suspension travel, shock absorbers (the latter are neglected in the static analysis, the damping forces depending on speeds).

The literature shows various works regarding the evaluation of the static and dynamic behaviour of the beam axle suspension systems, considering their importance for the comfort and stability

characteristics of the vehicles [3-6]. The research carried out in this paper continues a previous work by the author [7], which aimed to develop an analytical method for finding the static equilibrium position/configuration of the suspension system. This method is based on the principle of virtual mechanical work and integrates an original numerical algorithm for the positional analysis of the axle guidance mechanisms, which allows the determination of the deformations (and subsequently the reaction forces) in the elastic elements of the suspension. Knowing the static equilibrium position is important to assess the behaviour of the vehicle in various operating and loading regimes (such as stationary, traction, braking or skidding). The study in this paper focuses on determining the reaction forces and torques in the compliant elements of the suspension system. The amplitude of these elastic reactions is very important to be known for the proper sizing of the elastic elements, otherwise there is either a risk that these will be altered prematurely, or that the suspension will be too soft or rigid.

## 2. The static model of the axle suspension system

The static model of the beam axle suspension system considered in this work is that schematically shown in Figure 1. The wheels guidance mechanism of this suspension is coded 5-SS, considering that the mechanism consists of five bars, four of which (3, 4, 5, 6) are arranged longitudinally (to take the longitudinal forces from the contact between wheels and track), and one (so-called Panhard bar - 7) in the transversal plane (thus taking over the transverse forces).

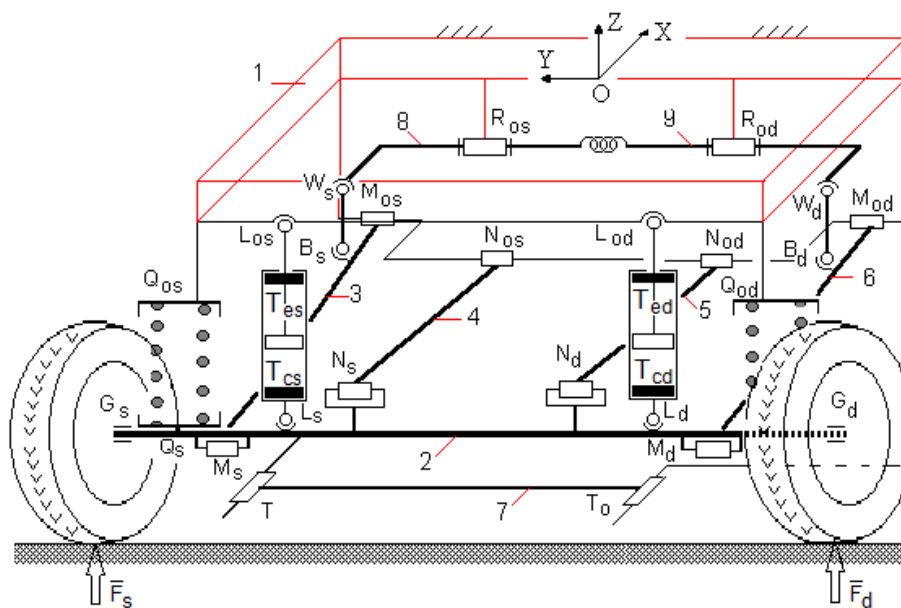


Figure 1. The static model of the 5-SS axle suspension system.

The left and right springs and shock absorbers are arranged vertically between axle ( $Q_{s/d}$ ,  $L_{s/d}$ ) and chassis ( $Q_{0s/d}$ ,  $L_{0s/d}$ ). The anti-roll/stabilizer bar is formed by two rigid semi-bars (8, 9), between which a torsion bar spring is arranged. The semi-bars are connected by revolute joints to the chassis, while their other end is articulated by connecting rods ( $W_{s/d}$   $B_{s/d}$ ) to the lower longitudinal bars of the guidance mechanism. Considering their small dimension/mass compared to the other bodies in the suspension system, with the purpose to simplify the theoretical model, the two rods are modeled by composed restrictions of type sphere-to-sphere between the adjacent bodies (anti-roll semi-bar – lower guide bar). The compression ( $T_{cs/d}$ ) and expansion ( $T_{es/d}$ ) bump-stops are mounted inside the shock absorbers, thus limiting the relative movement (up and down) of the piston relative to the cylinder. Unlike the dynamic model, in which the car body (chassis) is mobile, in the static model the chassis is fixed maintained, thus constituting the basis of the mechanism. As mentioned above, the reaction (damping) forces in the shock absorbers are not taken into account, as they depend on the speeds. The

values of the geometric and elastic parameters for the suspension in study were collected from the technical documentation of an existing off-road vehicle.

The suspension system considered in this study is characterized by the following: number of generalized coordinates for 8 mobile bodies (the axle, the guide bars of the 5-SS mechanism, the connecting rods of the anti-roll bar):  $8 \times 6 = 48$ ; number of degrees of freedom removed by the geometrical restrictions (i.e. revolute joints in  $R_{0s/d}$ ):  $2 \times 5 = 10$ ; number of degrees of freedom removed by the composed restrictions ( $W_{s/d} B_{s/d}$ ):  $2 \times 1 = 2$ . In these terms, the number of degrees of freedom (DOF) of the suspension system can be determined by applying the Gruebler count, as follows:  $DOF = 48 - (10 + 2) = 36$ .

### 3. Results and conclusions

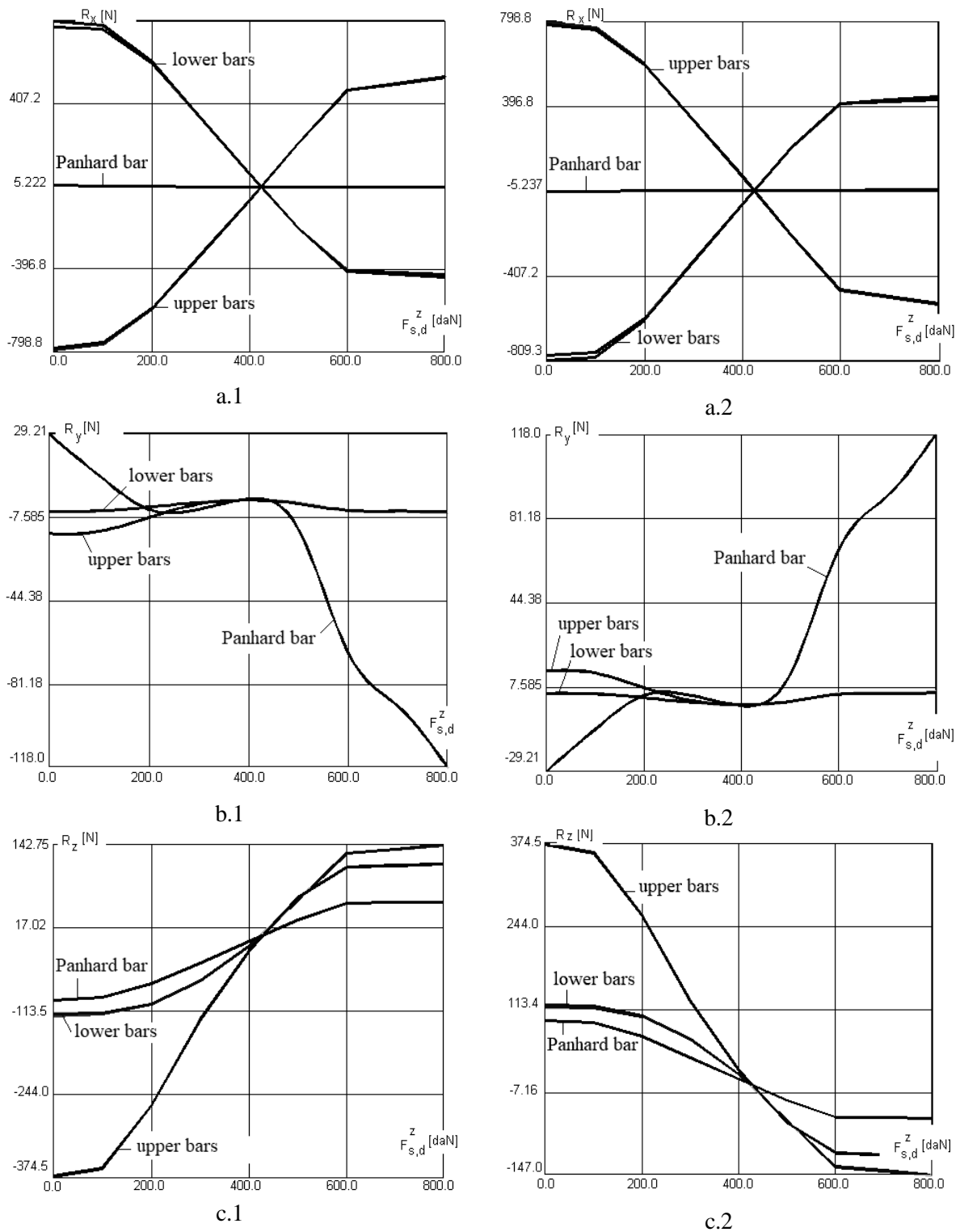
The external loading of the suspension system is achieved by the forces applied to the wheels ( $F_{s/d}$ ) at the point of contact with the ground, which are dependent on the specific operating regime of the vehicle. These reactions (longitudinal, transversal and vertical, as the case may be) are determined from the general dynamic balance of the vehicle, according to the procedure detailed in [7].

For beginning, the study is carried out by considering the stationary loading regime (which is one of the representative static regimes), where the external load of the suspension system is achieved by identical vertical forces at the left and right wheels ( $F_s^z = F_d^z = F_z$ ) in the range of values  $F^z \in [0, 800]$  daN, while the longitudinal and transversal forces are null. Given the range considered for the vertical forces at the wheels, it is actually a quasi-static analysis (i.e. a series of static simulations, for various values of the forces).

The results obtained by the quasi-static analysis of the 5-SS axle suspension system in the stationary regime are shown in Figures 2 (the reactions in bushings) and Figure 3 (the reactions in the other elastic elements of the suspension, namely springs, anti-roll bar, compression and expansion bump-stops). For the reaction forces in bushings, the following findings can be formulated:

- the main reaction is the longitudinal one (along X), the bushings of the lower and upper arms being loaded by approximately equal forces; the bushings of the Panhard bar practically do not take longitudinal reactions; greater reactions occur when the suspension is extended;
- the lateral/transversal reactions (along Y) are practically insignificant, with a certain increase in the reactions in the bushings of the Panhard bar when the suspension is compressed;
- significant vertical reactions (along Z) appear only in the bushings of the upper arms when the suspension is extended;
- when the suspension reaches the compression or expansion bumpers (as the case may be), a reduction of the reaction forces in bushings is observed, part of the mechanical work developed by the external forces on the wheels being consumed for the deformation of the bumpers;
- because the forces from the springs and bumpers act at the level of the axle and not on the guidance arms of the mechanism (these elastic elements being arranged between the axle and the car body), as well as due to the fact that the left & right external loads are identical, which makes the anti-roll bar not deform (so it does not introduce reaction forces), the reactions in the bushings from the car body and the axle are almost equal (and in opposite directions); however, there is a very small difference between the values of these reaction forces due to the arrangement mode of the Panhard bar, which results in a certain asymmetry in the kinematic scheme of the mechanism;
- under the conditions in which the bushings are mounted in the static loading position of the suspension mechanism (the initial data by which the mechanism was modeled correspond to this position), the reaction forces become zero for values of the vertical forces at wheels corresponding to rising the suspension mechanism in this position ( $F^z \cong 423.8$  daN).

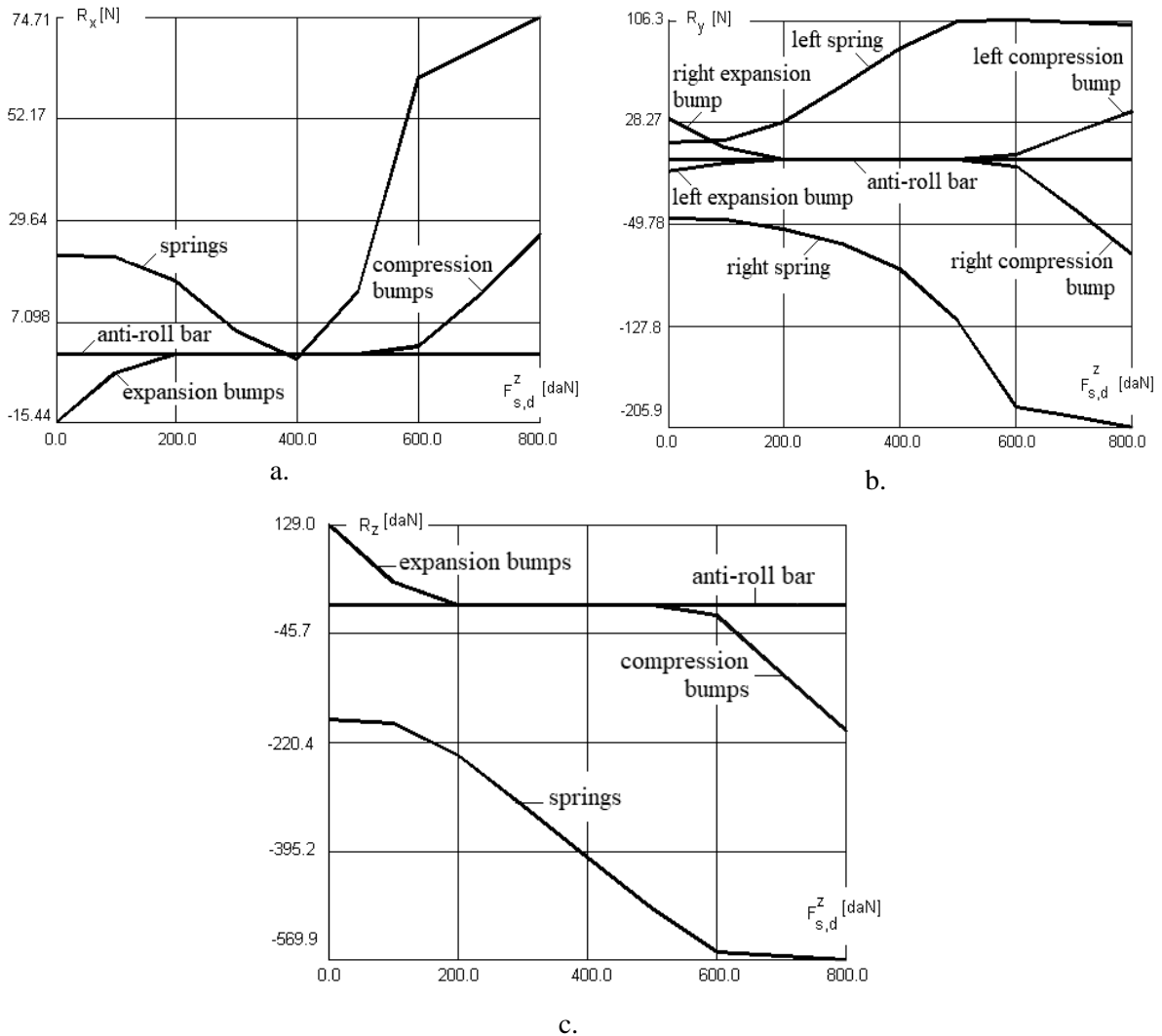
The reaction forces from the other elastic elements of the axle suspension system (springs, bump-stops, anti-roll bar) are those shown in Figure 3. Obviously, when the suspension reaches the bump-stops (so when compressing or expanding), the elastic forces in springs decrease. The reaction forces in the bump-stops are non-stationary, this type of elastic connection being a rheonomic one. As previously mentioned, in the stationary regime (with equal external forces applied to the left and right wheels), the anti-roll bar practically does not operate (so no reactions are generated in this elastic element).



**Figure 2.** The reaction forces in bushings for the stationary regime: a - longitudinal radial reactions; b - axial reactions, c - vertical radial reactions; 1 - bushings on car body, 2 - bushings on axle.

Regarding the elastic reaction torques, only the results obtained for the bushings on the car body are presented below (the torques in the bushings on the axle are very close to these). According to the diagrams in Figure 4, the following conclusions can be formulated:

- the main load is, obviously, the torsion of the bushings, the most affected/loaded being the bushings of the longitudinal bars;
- the conical torques in the horizontal radial direction are actually produced only in the bushings of the Panhard transverse bar, while the reaction torques around the vertical axis have relatively close values for all bushings.

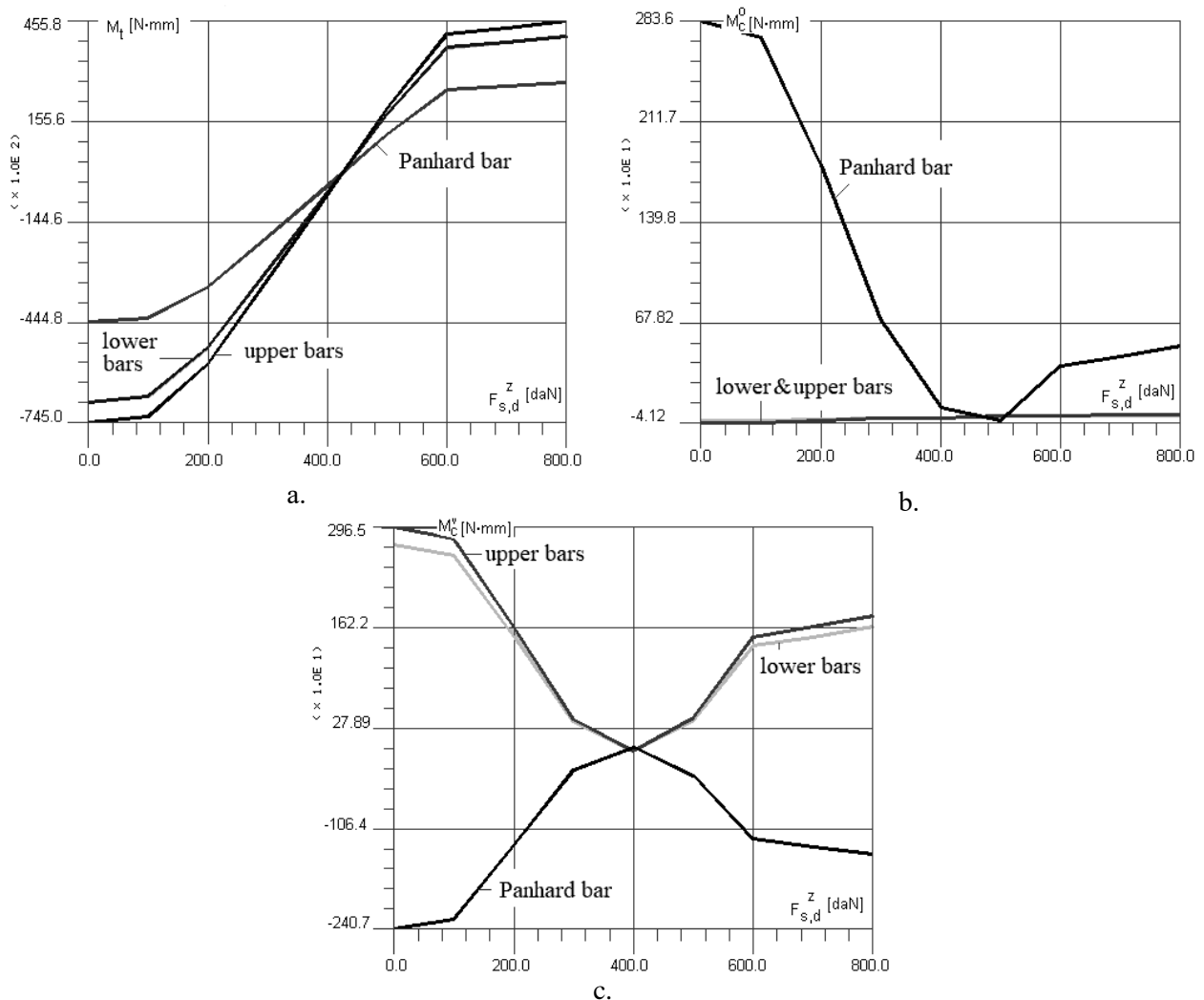


**Figure 3.** The reaction forces in springs, bump-stops and anti-roll bar: a - longitudinal reactions, b - lateral reactions, c - vertical reactions.

The second representative static case studied in this paper is that corresponding to the traction - braking regime. In this case, the external loading of the suspension system is made by constant vertical forces  $F_s^z = F_d^z = 423.8$  daN (corresponding to lifting the mechanism in the static load position), while the longitudinal forces  $F_s^x$  and  $F_d^x$  vary in the range of values  $F^x \in [-400, 400]$  daN. From the diagrams shown in Figure 5, the following conclusions can be drawn:

- the external longitudinal forces ( $R_x$ ) are mainly taken over by the bushings of the lower arms, but also (in a slightly smaller measure) by the bushings of the upper arms;
- the lateral/transversal reactions ( $R_y$ ) are very small (practically negligible) in the case of all bushings;
- the vertical reactions ( $R_z$ ) have low values compared to the longitudinal ones, loading the bushings of the lower and upper longitudinal arms to approximately the same extent;

- as in the case of the stationary regime, the forces in the bushings on the car body are equal and opposite to those in the bushings on the axle;
  - the suspension no longer "reaches" the bump-stops, therefore the mechanical work developed by the external forces on the wheels is consumed only for the deformation of the bushings and of the springs.
- It should be mentioned that for the Panhard bar's bushings, the axial reaction is oriented along the X axis, while the longitudinal radial reaction is in the direction of the Y axis.

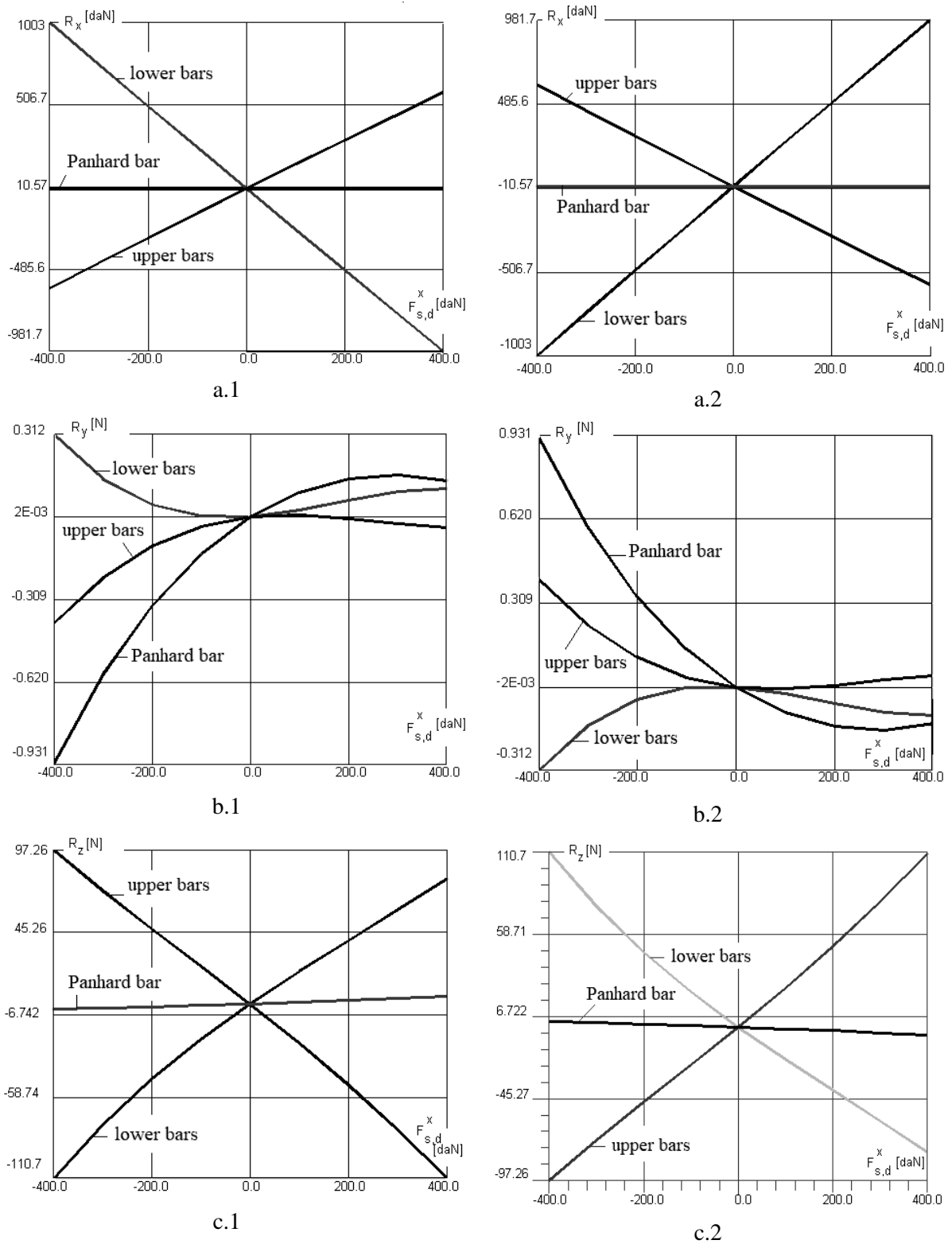


**Figure 4.** The elastic torques in bushings for the stationary regime: a - torsional torques, b - horizontal conical torques, c - vertical conical torques.

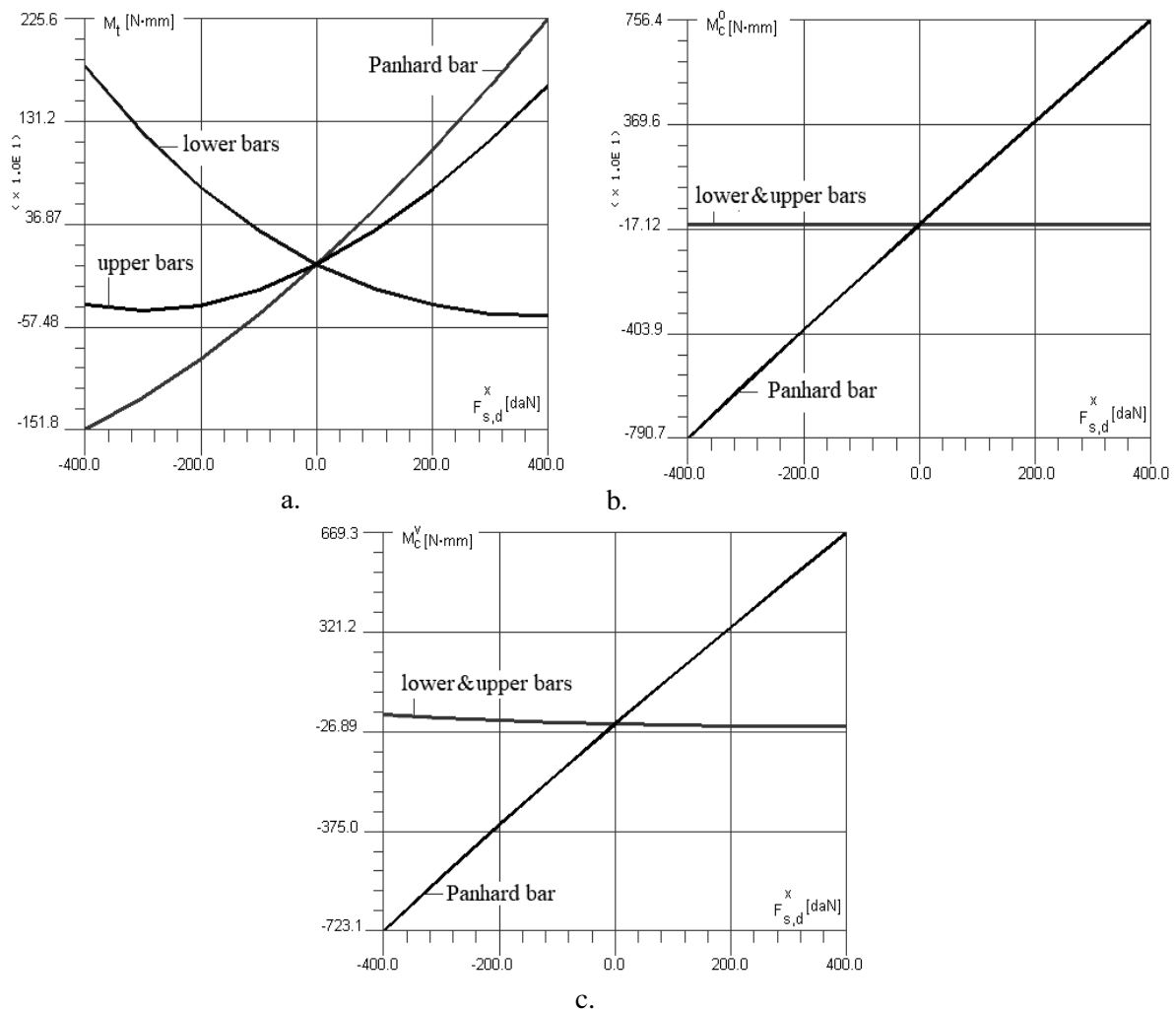
Regarding the elastic reaction torques in bushings, the results for the traction - braking regime are shown in Figure 6, from where the following conclusions can be formulated:

- the main load corresponds to the rotation around the horizontal axis of the Panhard bar's bushing;
- the bushings of the longitudinal bars practically support only torsions (of small values, however), as is the case with the Panhard bar;
- the reaction torques around the vertical axis have very small values, even for the bushing of the Panhard bar.

Such results, coupled with those obtained through the dynamic analysis (which will be presented in a future work), allow the accurate assessment of the behavior of the suspension system, which has a crucial role on the comfort and stability performance of the vehicle.



**Figure 5.** The reaction forces in bushings for the traction-braking regime: a - longitudinal radial reactions; b - axial reactions, c - vertical radial reactions; 1 - bushings on car body, 2 - bushings on axle.



**Figure 6.** The elastic torques in bushings for the traction-braking regime: a - torsional torques, b - horizontal conical torques, c - vertical conical torques.

## References

- [1] Alexandru C 2009 The kinematic optimization of the multi-link suspension mechanism used for rear axle of the motor vehicle *Proceedings of the Romanian Academy* **10**(3) pp 244-253
- [2] Alexandru P, Vișa I, Alexandru C 2014 Modeling the angular capability of the ball joints in a complex mechanism with two degrees of mobility *Applied Mathematical Modelling* **38**(23) pp 5456-5470
- [3] Ambrosio J, Verissimo P 2009 Sensitivity of a vehicle ride to the suspension bushing characteristics *Journal of Mechanical Science and Technology* **23**(4) pp 1075-1082
- [4] Knapczyk J, Maniowski M 2010 Optimization of 5-rod car suspension for elastokinematic and dynamic characteristics *The Archive of Mechanical Engineering* **52**(2) pp 133-147
- [5] Tică M, Dobre G, Mateescu V 2014 Influence of compliance for an elastokinematic model of a proposed rear suspension *International Journal of Automotive Technology* **15**(6) pp 885-891
- [6] Țoțu V, Alexandru C 2013 Study concerning the effect of the bushings' deformability on the static behavior of the rear axle guiding linkages *Applied Mechanics and Materials* **245** pp 132-137
- [7] Alexandru C 2019 Method for the quasi-static analysis of beam axle suspension systems used for road vehicles *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering* **233**(7) pp 1818-1833