

Practical solution for improving the mechanical behaviour of the composite materials reinforced with flax woven fabric

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Abstract

This article focuses on finding a practical solution for improving the mechanical behaviour especially in bending in case of the composite materials reinforced with flax fabric. For this purpose, two kinds of composites having the same number of layers are mechanically tested: the flax/epoxy composite and the flax/glass/epoxy composite (hybrid composite). The mechanical tests used were tensile test and bending test (three-point method). Different results corresponding to the weft or warp directions were remarked in case of the hybrid composite: for example, the maximum values of the tensile strength and the flexural strength measured in the weft direction are 40.63% and 34.61% greater, respectively, than the values corresponding to the warp direction. This article reports the improvement of the mechanical properties by replacing the flax reinforcement with glass reinforcement in the two upper and lower layers of the flax composite materials: increasing with 35.19% in terms of Young's modulus E recorded in the tensile test, with 79.86% of the maximum value of the tensile stress and with 91.08% of the maximum value of the flexural stress. Composite beam theory was considered and theoretical approaches were validated by the experimental results.

Keywords

Hybrid, composite material, flax fibres, mechanical testing

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Introduction

Statistics over the last decade shows an increasing interest for manufacturing of the plastic composites reinforced with natural fibres: flax,^{1,2} hemp,^{2–4} jute^{2,5} and wood fibres.⁶ Statistics shows 4,275,049 ha¹ of harvested area of flax fibres during 2001–2011. France and Belarus each held 26% of the total harvested area of flax fibres in 2011¹ while Russian Federation and China held 20% and 14% of the total harvested area, respectively.

The applications of the composite materials reinforced with flax fibres include^{1,2} automotive interior components (such as door and panels of interior design), roof panels, insulation wall panels, boat hulls,¹

tennis rackets, surfboards,¹ packaging boxes, chairs¹ and other furniture parts, and wind turbine blade.⁷

The major advantages of such natural fibres consist in the following: these represent a renewable source⁵ and provide lower weight to the new composites due to their lower densities compared to the classical

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Table 1. Comparison between the natural fibres and other fibres in terms of density.

Type of fibres	Density ρ (kg/dm ³)	References
Flax fibres	1.4–1.52	Müssig and Haag, ¹ Müssig ²
Hemp fibres	1.4–1.6	Dhakar and Zhang ³
Jute fibres	1.3–1.5	Müssig and Haag, ¹ Müssig ²
Cotton fibres	1.5–1.6	Müssig and Haag, ¹ Müssig ²
Glass fibres	2.16–2.54	Alamoreanu and Chirita, ⁹ Soler ¹⁰
Carbon fibres	1.74–1.96	Alamoreanu and Chirita, ⁹ Soler ¹⁰
Kevlar fibres	1.45–1.47	Soler ¹⁰
Boron fibres	2.7	Soler ¹⁰
Banana fibres	1.35	Dhakar and Zhang ³
Wood		
Fir	0.374	Curtu and Ghelmeziu ¹¹
Spruce	0.4	
Oak	0.7	
Beech	0.618	
Walnut	0.6	

reinforcement fibres (glass, carbon, boron and Kevlar fibres) as shown in Table 1. The flax fibres are recommended for insulation panels since the sound absorption coefficient recorded in case of the flax/Araldide composite is 21.42%⁸ greater than the corresponding value recorded in case of the E-glass/Araldide composite.

But natural fibres have some major disadvantages: these absorb water^{3,6,12,13} in wet environments and degrade in such environments and under the action of ultraviolet (UV) light;¹³ smoke emission is high when firing.^{1,6} The water absorbed (13.5% weight) by the flax/epoxy composite until saturation was 12.85 times greater than the water absorbed (1.05%) by the glass/epoxy composite.¹² In the case of the flax fabric/epoxy composites, the accelerated UV weathering tests for 1500 h by ageing cycles (exposing to Fluorescent UV light at 60°C and water spraying) lead to the decreasing with 29.9% and 34.9%¹³ of the tensile strength and tensile modulus, respectively. After similar weathering tests, the flexural properties also decrease, with 10% in case of the flexural strength and with 10.2% in case of the flexural modulus.¹³

Many applications of the composite materials reinforced with flax fibres refer to randomly reinforcing with chopped flax fibres,¹ to reinforcing with continuous flax fibres by using biaxial woven fabric⁷ or to reinforcing with continuous unidirectional fibres.^{12–14}

Comparative analysis was made¹⁴ regarding the results obtained in case of composite materials based on epoxy resin reinforced with natural fibres (flax, jute, sisal, hemp). The tensile modulus corresponding to the flax/epoxy composite was reported to be 152.49%, 102.35% and 182.35%¹⁴ greater than that corresponding to the composite materials reinforced with jute, sisal and hemp fibres, respectively.

Recent articles^{15,16} revealed some ways to improve the mechanical properties of the composite materials

reinforced with flax fibres. For example, Xue and Hu¹⁵ showed that NaOH treatment applied to the flax reinforcement leads to the improved mechanical properties of such composite materials. Another work¹⁶ also showed that the interfacial bonding between fibres and polymeric matrix can be improved by chemical treatments (with maleic anhydride, vinyl-trimethoxy silane, maleic anhydride-polypropylene copolymer) applied to the matrix.

A recent paper⁷ has tried to replace E-glass fibres with flax fibres in structural composites. For this purpose, the authors had investigated two variants of the 3.5-m composite rotor blades: one made of flax/polyester and the other one manufactured from E-glass/polyester. It could be remarked that the flax/polyester blade is 10% lighter than the E-glass/polyester blade; both the versions satisfy the design and structural integrity requirements for an 11-kW turbine according to the certification standards. But the E-glass/polyester blade was stiffer because the maximum deflection recorded in case of the flax/polyester blade was greater than in case of the E-glass/polyester blade.

Some experimental investigations (tensile and compression tests) were conducted on flax/epoxy and glass/epoxy composites¹⁷ made of the same epoxy matrix system SR 8200/SD 8205 and containing the same 43% fibre volume ratio. It was shown that the ultimate tensile stress, tensile modulus E and ultimate compression stress corresponding to the glass/epoxy composite were 123.53%, 50.03%, 76% greater, respectively, than the ones measured for the flax/epoxy composite.

The literature does not report results on hybrid laminated composites reinforced with both flax and glass woven fabrics made of continuous flax/glass fibres. This is the reason why the present work proposes a hybrid solution of the composite material reinforced with both flax and glass fabric. The novel hybrid composite should combine the advantages of both the

flax fibres and the glass fibres. The advantages of the flax fibres have already been mentioned above. The advantages of glass fibres are good ratio between strength and weight, low cost, these are not hydrophilic; these are alkaline resistant. Glass fibres have the tensile strength of 3515 MPa⁹ greater than the value of 700 MPa recorded in the case of flax fibres.¹

In order to determine which is the better solution, the mechanical properties of the hybrid composite material are compared with the ones corresponding to the composite material reinforced only with flax fibres.

On the other hand, a theoretical model of the beam theory is approached for both the composite materials involved in this study: flax/epoxy composite and glass/flax/epoxy composite. First of all, the beam theory used in the case of laminated composite materials is presented taking into account the literature^{18–20} in order to compute both the equivalent modulus of elasticity E_x of the composite beam and the maximum deflection $w_{0 \max}$ of the cross section located at the midpoint of the beam. Then, the relation of the equivalent modulus of elasticity E_x is particularized for the particular cases of the specimens made of flax/epoxy or glass/flax/epoxy composites, subjected to bending by the three-point method.

Finally, the theoretical results obtained by using the beam theory of the laminated composite are compared with the experimental results obtained in bending tests.

Materials and work method

Structure of the composite materials tested

In this investigation, two kinds of composite materials are tested. The first composite material contains eight layers made of Epolam 2015 epoxy resin reinforced only with flax woven fabric. The other composite material is a hybrid one because some layers are made of Epolam 2015 epoxy resin reinforced with flax woven fabric, while the others are reinforced with glass woven fabric. The content of the fibres was equal to 40 wt%. Figure 1 shows the layout of the layers of the hybrid composite material.

The densities of the woven fabrics used are $\rho = 280 \text{ g/m}^2$ in the case of flax woven fabric and $\rho = 200 \text{ g/m}^2$ in the case of glass woven fabric. It is known that different kinds of flax yarns were used in the weft and warp directions to manufacture the flax woven fabric. This aspect had to be taken into account when preparing specimens. On the contrary, in the case of glass woven fabric, glass yarns are not different in the weft and warp directions.

First, a board ($600 \times 350 \text{ mm}^2$) made of each composite material was manufactured by using hand lay-up technology. The thickness of the flax/epoxy composite board was equal to $5.4 \pm 0.1 \text{ mm}$, while the thickness of

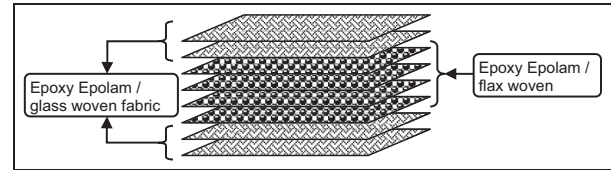


Figure 1. Laminated structure of the glass/flax hybrid composite material.

the hybrid composite board was equal to $3.9 \pm 0.1 \text{ mm}$. Vacuum equipment was used to eliminate air gaps. The conditioning time for the both plates was 2 weeks at room temperature.

Then, the plates made were cut to obtain specimens for tensile test according to SR EN ISO 527-4:2000;²¹ the dimensions of the rectangular specimens for bending were $15 \times 100 \text{ mm}^2$ according to SR EN ISO 14125:2000.²²

In the case of each composite material tested, two sets of specimens were manufactured for each mechanical test: a set of specimens whose length is parallel to the weft direction of the flax fabric, while the other set contains specimens whose length is parallel to the warp direction. This consideration was made by taking into account that the flax yarns used for weft and warp directions are different according to the technical sheet of the flax woven fabric.

Mechanical testing

The testing equipment LR5K Plus machine manufactured by LLOYD Instruments used for both tensile test and flexural test, consists of hydraulic power supply. The maximum force capacity is $\pm 15 \text{ kN}$. The speed of loading was equal to 1 or 1.5 mm/min during the tensile test or during the bending, respectively. The three-point method was used for testing in bending (flexural test).

Before each mechanical test of a specimen, the dimensions of the cross section were accurately measured, and then, they were considered as input data in the software program of the machine.

The testing equipment allowed us to record pairs of values in the form of text files having 200–300 lines: tensile force F and extension of the specimen; bending force F and deflection v at the midpoint of the flexural specimen. The experimental data were statistically processed in order to determine the mechanical properties. The modulus of elasticity E in the both tensile test and bending test was determined on the linear portion of the loading curve. Therefore, the average values of the following quantities could be accurately computed for tensile test: Young's modulus E in tensile test; maximum tensile stress σ_{\max} ; elongation Δl to the maximum force F_{\max} ; normal strain ε to the maximum force

F_{\max} and mechanical work W done until the maximum force F_{\max} .

The average values determined in the bending test are as follows: Young's modulus E in flexural test; flexural rigidity EI_z ; maximum bending stress σ_{\max} at the maximum load and mechanical work W done until the maximum force F_{\max} .

Theoretical approaches

Beam theory

All layers of the composite materials analysed are reinforced with bidirectional woven fabric made of flax or glass fibres. Moreover, the layers made of the same material and with the same thickness are symmetrically placed with respect to the median surface of the composite material. It follows that both composite materials tested are symmetric special orthotropic laminated composite materials.

The general constitutive equation^{18,19} of the plate element made of laminated composite material is:

$$\begin{pmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{pmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{pmatrix} \quad (1)$$

where $[A]$ represents the rigidity matrix in plane, $[B]$ is the bending-tension coupling matrix and $[D]$ is the rigidity matrix in bending.

In the case of a plate element made of symmetric special orthotropic laminated composite (Figure 2), subjected just to the bending, the constitutive equation (1) is reduced to:

$$\begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{pmatrix} \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{pmatrix} \quad (2)$$

because $B_{ij} = 0 (i, j = 1, 2, 6)$ in the case of the symmetric laminated composite and $D_{16} = D_{26}$ in the case of the special orthotropic laminated composites.

In relation (2), the curvatures are computed by using the following relations:

$$\begin{aligned} \kappa_x(x, y) &= -\frac{\partial^2 w_0}{\partial x^2}; \quad \kappa_y(x, y) = -\frac{\partial^2 w_0}{\partial y^2}; \\ \kappa_{xy}(x, y) &= -2\frac{\partial^2 w_0}{\partial x \partial y} \end{aligned} \quad (3)$$

where w_0 is the displacement of the arbitrary point of the plate element along the Oz axis.

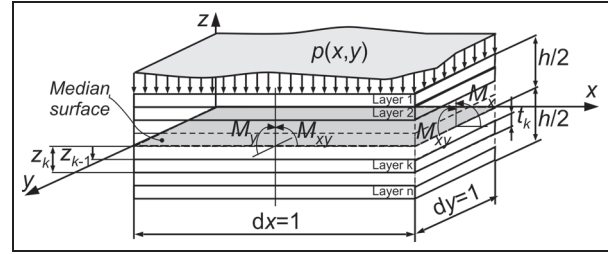


Figure 2. Internal forces developed at the level of the median surface of a plate element made of laminated composite material.

In relation (2), the terms of the bending rigidity matrix $[D]$ are computed by using the relation:

$$D_{ij} = \frac{1}{3} \sum_{k=1}^N (\overline{Q}_{ij})_k (z_k^3 - z_{k-1}^3), \quad i, j = 1, 2, 6 \quad (4)$$

where $(\overline{Q}_{ij})_k$ are the terms of the rigidity matrix $[\overline{Q}]$ of an arbitrary layer k with respect to the global coordinate system $xOyz$. The rigidity matrix $[\overline{Q}]$ links the stress vector $\{\sigma\}_k$ computed at the level of the arbitrary point located in the layer k and the strain vector $\{\varepsilon\}_k$ computed for the same point, with respect to the global coordinate system $xOyz$: $\{\sigma\}_k = [\overline{Q}]_k \cdot \{\varepsilon\}_k$.

Since the orientations of the fabrics are the same (0°) in all layers of the composite materials analysed, the rigidity matrix $[\overline{Q}]$ of an arbitrary layer k with respect to the global coordinate system $xOyz$ is the same with the rigidity matrix $[Q]$ computed with respect to the local coordinate system called material coordinate system whose axes 1 and 2 are aligned with the weft direction and warp direction of the woven fabric, respectively:

$$[\overline{Q}]_k = [Q]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} = \begin{bmatrix} \frac{E_1}{\Delta} & \frac{\nu_{12}E_2}{\Delta} & 0 \\ \frac{\nu_{12}E_2}{\Delta} & \frac{E_2}{\Delta} & 0 \\ 0 & 0 & G_{12} \end{bmatrix} \quad (5)$$

where $\Delta = 1 - \nu_{12} \cdot \nu_{21}$. Poisson's ratio ν_{21} may be computed by using the relation between the elastic constants:^{18,19}

$$\frac{\nu_{21}}{E_2} = \frac{\nu_{12}}{E_1} \quad (6)$$

Relation (2) may be written in the following form:

$$\begin{pmatrix} \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{pmatrix} = [D]^{-1} \begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} = \begin{bmatrix} \delta_{11} & \delta_{12} & 0 \\ \delta_{12} & \delta_{22} & 0 \\ 0 & 0 & \delta_{66} \end{bmatrix} \begin{pmatrix} M_x \\ M_y \\ M_{xy} \end{pmatrix} \quad (7)$$

where δ_{ij} are the components of the inverse matrix of $[D]$.

The relations between the vector of moments $\{M_x \ M_y \ M_{xy}\}^T$ developed at the level of the median surface of the composite plate element and the vector of the stresses $\{\sigma_x \ \sigma_y \ \tau_{xy}\}^T$ developed at the level of the arbitrary cross section:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \cdot z \, dz \quad (8)$$

lead to the following relation:^{18,19}

$$\begin{Bmatrix} \kappa_x^0 \\ \kappa_y^0 \\ \kappa_{xy}^0 \end{Bmatrix} = \frac{12}{h^3} \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} \quad (9)$$

where h is the thickness of the composite plate (Figure 2); E_x , E_y , ν_{xy} are the elastic characteristics of the fictitious plate made of an orthotropic material, equivalent to the composite, material having a symmetric special orthotropic laminated structure.

In the case of beam theory,¹⁹ it is assumed that the bending moment M_y and the twisting moment M_{xy} are equal to 0 (Figure 3):

$$M_y = 0; \ M_{xy} = 0 \quad (10)$$

Thus, using the first relation from equation (3), relation (7) leads to:

$$\kappa_x = -\frac{\partial^2 w_0}{\partial x^2} = \delta_{11} M_x \quad (11)$$

and relation (9) is reduced to:

$$E_x = \frac{12}{h^3 \delta_{11}} = \frac{12(D_{11}D_{22} - D_{12}^2)}{h^3 D_{22}} \quad (12)$$

Relation (11) is usually written as follows by using relation (12):

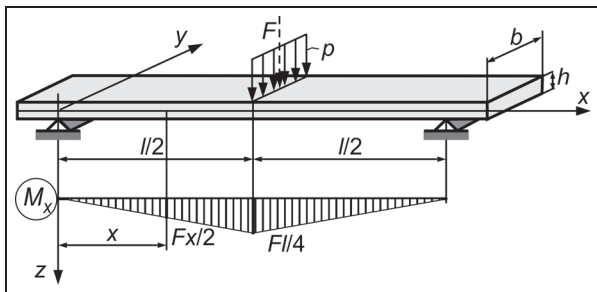


Figure 3. Composite beam subjected to bending by the three-point method.

$$\frac{d^2 w_0}{dx^2} = -\frac{M}{E_x I} \quad (13)$$

where $M = b \cdot M_x$ is the bending moment developed at the level of the arbitrary cross section of the beam and $I = bh^3/12$ is the axial moment of inertia of the beam cross section with respect to the axis Oy (Figure 3).

Considering the beam shown in Figure 3 that is subjected to the three-point bending, the differential equation (13) of the deformed median fibre of the beam may be written in the following form:

$$\frac{d^2 w_0}{dx^2} = -\frac{Fx}{2E_x I} \quad (14)$$

where $M = -Fx/2$ is the bending moment developed at the level of the arbitrary cross section located at a distance x with respect to the simple support.

The boundary conditions of the beam (Figure 3) are:

$$\begin{cases} -\text{for } x = 0; & w_0 = 0 \text{ and } M = 0 \left(\frac{d^2 w_0}{dx^2} = 0 \right) \\ -\text{for } x = l/2; & \frac{dw_0}{dx} = 0 \end{cases} \quad (15)$$

where the condition written for $x = l/2$ shows that the tangent to the deformed median fibre of the beam is a horizontal line.

By integrating relation (14) and considering the above boundary condition (15), the vertical deflection w_0 of the arbitrary cross section may be computed by using the formula:

$$w_0 = -\frac{Fl^2}{48E_x I} x \left[3 - \left(\frac{2x}{l} \right)^2 \right] \quad (16)$$

The maximum deflection $w_{0 \max}$ of the cross section located at the midpoint of the beam ($x = l/2$) is computed by using the following formula:

$$w_{0 \max} = \frac{Fl^3}{48E_x I} \quad (17)$$

where $E_x = 12/h^3 \delta_{11}$ according to relation (12).

Particular cases

Herein, the components of the rigidity matrix in bending $[D]$ corresponding to each composite material involved in this work were calculated in order to replace them in relation (12) of the theoretical value of the modulus of elasticity E_x corresponding to the specimen subjected to the bending (three-point method).

For this purpose, both the laminated composite structures are represented in Figure 4. The thickness and the coordinates (z_{k-1}, z_k) of each lamina are also shown in Figure 4.

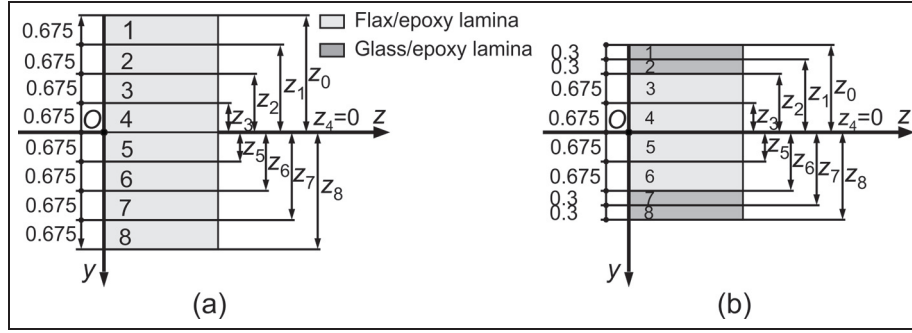


Figure 4. Thickness and coordinates of the layers in the case of two laminated composites analysed: (a) the flax/epoxy composite and (b) the glass/flax/epoxy hybrid composite.

The components of the rigidity matrix in bending $[D']$ corresponding to the flax/epoxy composite material are computed by using relation (4):

$$\begin{aligned} D'_{11} &= \frac{1}{3} \sum_{k=1}^8 (\overline{Q}_{11})_k (z_k^3 - z_{k-1}^3) \\ &= \frac{1}{3} \overline{Q}_{11} \left\{ [(-2.025)^3 - (-2.7)^3] \right. \\ &\quad + [(-1.35)^3 - (-2.025)^3] \\ &\quad + [(-0.675)^3 - (-1.35)^3] \\ &\quad + [0^3 - (-0.675)^3] + (0.675^3 - 0^3) \\ &\quad + (1.35^3 - 0.675^3) \\ &\quad \left. + (2.025^3 - 1.35^3) + (2.7^3 - 2.025^3) \right\} \\ &= 13.122 \overline{Q}_{11} \end{aligned} \quad (18a)$$

$$D'_{22} = \frac{1}{3} \sum_{k=1}^8 (\overline{Q}_{22})_k (z_k^3 - z_{k-1}^3) = 13.122 \overline{Q}_{22} \quad (18b)$$

$$D'_{12} = \frac{1}{3} \sum_{k=1}^8 (\overline{Q}_{12})_k (z_k^3 - z_{k-1}^3) = 13.122 \overline{Q}'_{12} \quad (18c)$$

where \overline{Q}_{ij} are the terms of the rigidity matrix $[\overline{Q}]$ corresponding to the flax/epoxy composite lamina with respect to the global coordinate system $xOyz$.

Relations (18) are replaced into relation (12) in order to obtain the theoretical formula of the modulus of elasticity E'_x corresponding to the bending specimen made of flax/epoxy composite material as follows:

$$\begin{aligned} E'_x &= \frac{12(D_{11}D_{22} - D_{12}^2)}{h^3 D_{22}} \\ &= \frac{12 \left[13.122^2 \overline{Q}_{11} \cdot \overline{Q}_{22} - (13.122 \overline{Q}'_{12})^2 \right]}{5.4^3 \cdot 13.122 \overline{Q}'_{22}} \\ &= \frac{\overline{Q}_{11} \cdot \overline{Q}_{22} - \overline{Q}'_{12}{}^2}{\overline{Q}'_{22}} \end{aligned} \quad (19)$$

where $h = 5.4$ mm (Figure 4(a)) represents the total thickness of the flax/epoxy composite structure.

In the same manner, the components of the rigidity matrix in bending $[D'']$ corresponding to the glass/flax/epoxy composite material are computed by using relation (4):

$$\begin{aligned} D''_{11} &= \frac{1}{3} \sum_{k=1}^8 (\overline{Q}_{11})_k (z_k^3 - z_{k-1}^3) \\ &= \frac{1}{3} \left\{ \overline{Q}''_{11} [(-1.65)^3 - (-1.95)^3] \right. \\ &\quad + \overline{Q}'_{11} [(-1.35)^3 - (-1.65)^3] \\ &\quad + \overline{Q}_{11} [(-0.675)^3 - (-1.35)^3] \\ &\quad + \overline{Q}_{11} [0^3 - (-0.675)^3] + \overline{Q}_{11} (0.675^3 - 0^3) \\ &\quad + \overline{Q}'_{11} (1.35^3 - 0.675^3) + \overline{Q}''_{11} (1.65^3 - 1.35^3) \\ &\quad \left. + \overline{Q}'_{11} (1.95^3 - 1.65^3) \right\} \\ &= 3.303 \overline{Q}''_{11} + 1.64025 \overline{Q}'_{11} \end{aligned} \quad (20a)$$

$$D''_{22} = \frac{1}{3} \sum_{k=1}^8 (\overline{Q}_{22})_k (z_k^3 - z_{k-1}^3) = 3.303 \overline{Q}''_{22} + 1.64025 \overline{Q}'_{22} \quad (20b)$$

$$D''_{12} = \frac{1}{3} \sum_{k=1}^8 (\overline{Q}_{12})_k (z_k^3 - z_{k-1}^3) = 3.303 \overline{Q}''_{12} + 1.64025 \overline{Q}'_{12} \quad (20c)$$

where \overline{Q}_{ij} and \overline{Q}''_{ij} ($i, j = 1, 2$) are the terms of the rigidity matrix corresponding to the flax/epoxy composite lamina and to the glass/epoxy composite lamina, respectively, with respect to the global coordinate system $xOyz$.

Relations (20) are substituted into relation (12) in order to obtain the theoretical formula of the modulus of elasticity E''_x corresponding to the bending specimen made of glass/flax/epoxy composite material as follows

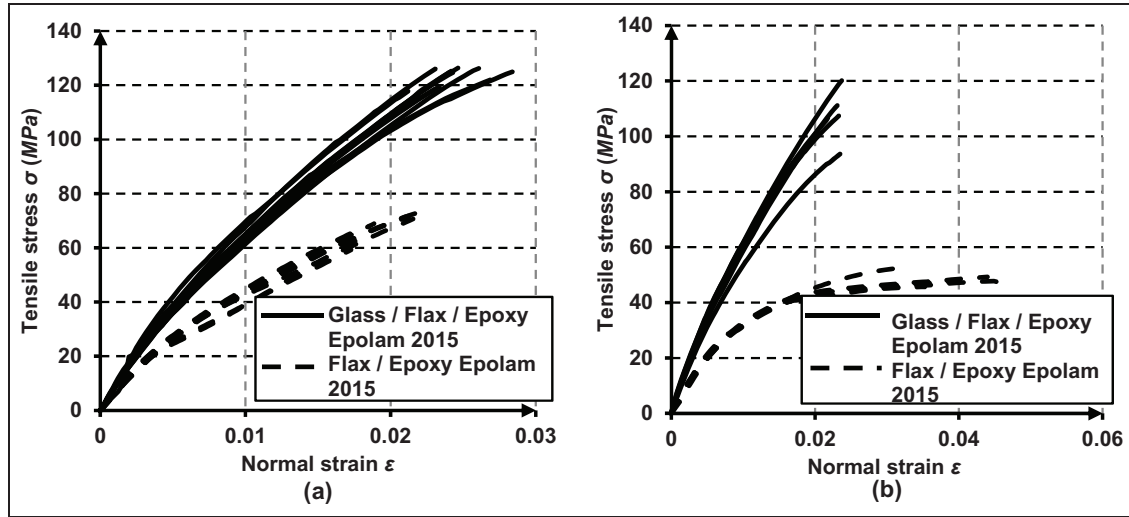


Figure 5. Stress–strain (σ – ε) curves: (a) weft direction and (b) warp direction.

$$E_x'' = \frac{12 \left[(3.303\overline{Q}_{11}'' + 1.64025\overline{Q}_{11}'') (3.303\overline{Q}_{22}'' + 1.64025\overline{Q}_{22}'') - (3.303\overline{Q}_{12}'' + 1.64025\overline{Q}_{12}'')^2 \right]}{3.9^3 \cdot (3.303\overline{Q}_{22}'' + 1.64025\overline{Q}_{22}'')} \quad (21)$$

where $h = 3.9$ mm (Figure 4(b)) represents the total thickness of the glass/flax/epoxy composite structure.

Finally, the theoretical values calculated by using relations (19) and (21) were compared with the experimental results in order to validate the theoretical models of the beam corresponding to both the composite materials involved in this work.

Results and comparisons: discussions

Experimental results

Figure 5 shows the stress–strain (σ – ε) curves recorded in tensile test in the case of both the composite materials involved in this study. Figure 5(a) presents the stress–strain curves corresponding to the case when the tensile force is parallel to the weft direction of the flax woven fabric used for reinforcing (length of the tensile specimen is parallel to the weft). On the other hand, Figure 5(b) shows comparative stress–strain curves recorded in the case of applying the tensile force in the warp direction.

In both the cases of tensile loading, it may be remarked that the stress–strain curves corresponding to the flax/epoxy composite material are located below the ones recorded in case of the hybrid composite material (glass/flax/epoxy composite). This means that the stiffness of the hybrid composite material is greater than the stiffness corresponding to the composite material reinforced only with flax woven fabric.

Figure 6 comparatively shows difference concerning the mechanical behaviour in the tensile test in case of

the two composite materials analysed in terms of tensile strength (Figure 6(a)) and Young's modulus E (Figure 6(b)).

Regarding the flax/epoxy composite material, it may be observed that the maximum value of the tensile stress σ_{\max} is 40.63% greater (Figure 6(a)) in the weft direction (68.29 MPa) than the value corresponding to the warp direction (48.56 MPa); Young's modulus E recorded in the tensile test is 33.84% greater (Figure 6(b)) in the weft direction (5870.64 MPa) than the one recorded in the warp direction (4386.22 MPa).

In the same manner, the results obtained in the case of the hybrid composite (glass/flax/epoxy) lead to the following remarks: the maximum value of the tensile stress σ_{\max} is 17.32% greater (Figure 6(a)) in the weft direction (122.83 MPa) than the value corresponding to the warp direction (104.69 MPa); Young's modulus E recorded in the tensile test is 9.29% greater (Figure 6(b)) in the weft direction (7936.45 MPa) than the one recorded in the warp direction (7261.58 MPa).

It could also be observed that the tensile properties measured in the weft direction increase in the case of hybrid composite with respect to the ones corresponding to the flax/epoxy composite as follows: with 35.19% in case of Young's modulus E and with 79.86% for the maximum value of the tensile stress σ_{\max} . Tensile properties measured in the warp direction of the hybrid composites are better again: Young's modulus E increased with 65.55% and the maximum value of the tensile stress σ_{\max} increased with 115.58%.

The other tensile properties of the composite materials tested are shown in Table 2.

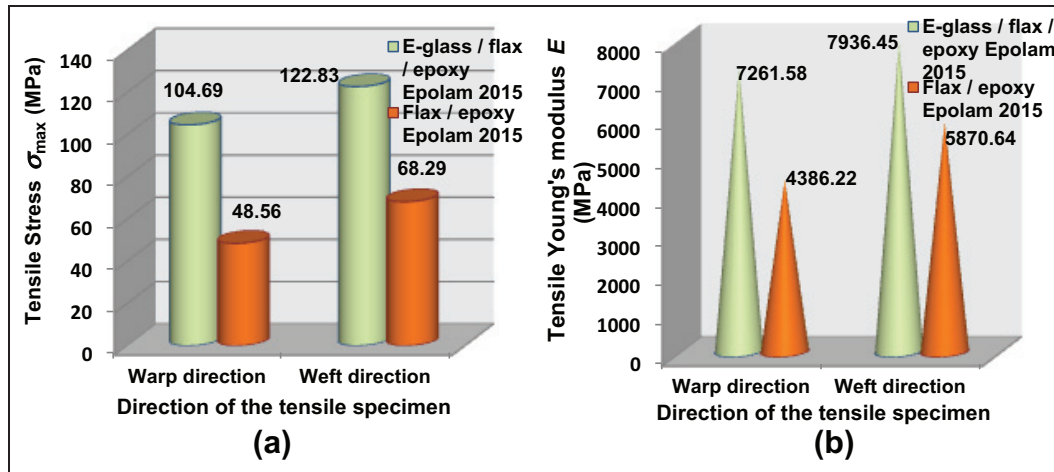


Figure 6. Comparison of the tensile properties in case of the tested composite materials: (a) tensile strength and (b) Young's modulus in tensile.

Table 2. The average values of the mechanical characteristics measured in the tensile test.

Composite material	Direction of the axial force	F_{max} (kN)	Elongation Δl to F_{max} (mm)	Strain ε to F_{max}	Work to maximum load (N mm)
E-glass/flax/epoxy Epolam 2015	Warp direction	4.104	1.141	0.0229	2727.88
	Weft direction	4.607	1.221	0.0244	2894.77
Flax/epoxy Epolam 2015	Warp direction	2.561	1.96	0.0400	4778.47
	Weft direction	3.715	0.99	0.0200	3541.46

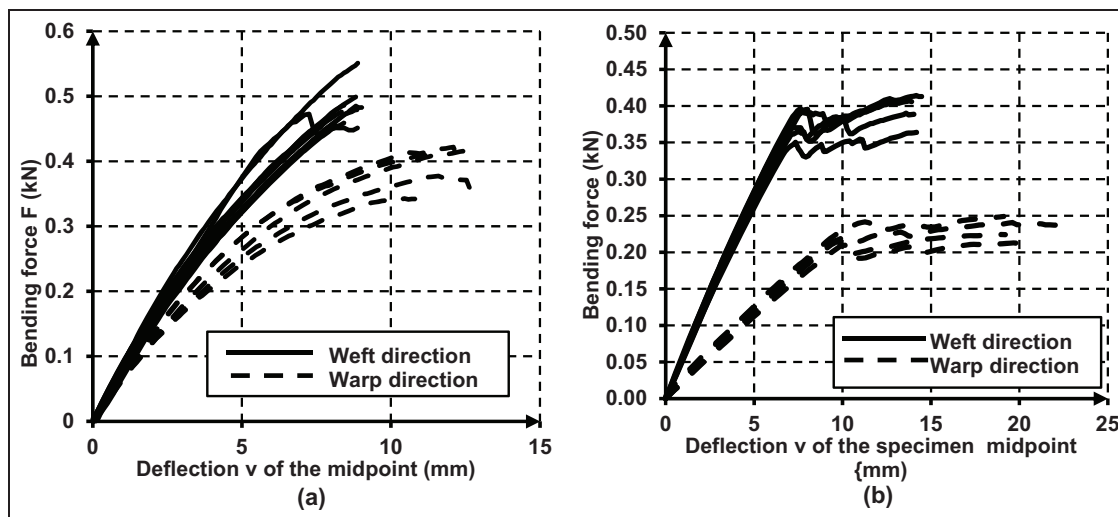


Figure 7. Force–deflection ($F-v$) curves in bending: (a) flax/epoxy composite and (b) glass/flax/epoxy.

Figure 7 shows the force–deflection ($F-v$) curves recorded during bending tests in case of both composite materials tested. It may be remarked again that the force–deflection curves ($F-v$) corresponding to the flexural specimens whose length is parallel to the weft direction are located above the ones that are parallel to the warp direction. This remark leads to the conclusion

that the specimens cut parallel to the weft direction are stiffer than the ones cut parallel to the warp direction.

Figure 8 shows the comparative mechanical properties determined in the bending test in terms of the maximum bending stress σ_{max} (Figure 8(a)) and Young's modulus E (Figure 8(b)).

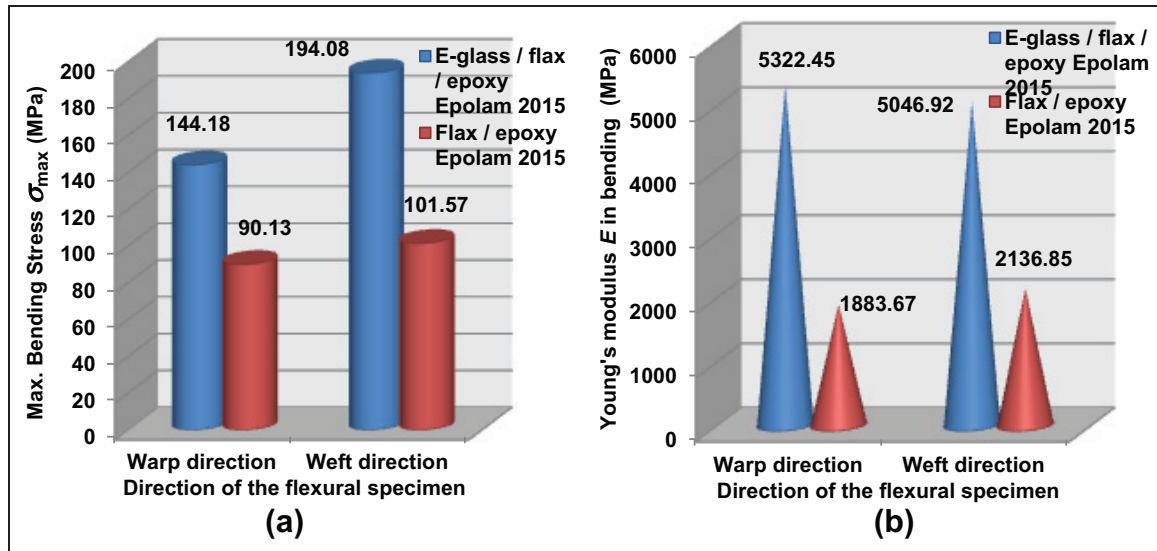


Figure 8. Comparison of the properties determined in the bending test in case of the tested composite materials: (a) maximum bending stress and (b) Young's modulus in bending.

Table 3. The average values of the mechanical characteristics measured in the flexural test.

Composite material	Direction of the length of specimen	Maximum load (N)	Work to break (N mm)
Flax/epoxy Epolam 2015	Warp direction	391.5	3226.8
	Weft direction	481.3	2340.8
E-glass/flax/epoxy Epolam 2015	Warp direction	231.65	3489
	Weft direction	454.86	4354

Regarding the flax/epoxy composite material, an increase was recorded for the mechanical properties corresponding to the weft direction with respect to the ones recorded in the warp direction: with 12.69% in the case of the maximum bending stress σ_{max} and with 13.44% in the case of Young's modulus E in bending.

In the case of the hybrid composite material, the average value of the maximum bending stress σ_{max} (Figure 8(a)) is 34.61% greater when the length of the flexural specimens is parallel to the weft direction with respect to the value corresponding to the specimens cut parallel to the warp direction. On the contrary, Young's modulus E in bending in the weft direction is 5.17% lower with respect to the value recorded in the warp direction. The flexural modulus values recorded in the two directions differ very little indeed. The explanation might be that the flax reinforcement layers are core layers. It is well known that in bending, the strain is linearly distributed over the thickness of the composite material, while the stress is linearly distributed over the layer thickness. Thus, it is noticed that the cutting direction of the specimen, in the weft or warp direction of the flax fabric, does not affect so much the modulus of elasticity E because the flax-reinforced layers are core layers.

In case of the hybrid composite, it could also be observed that the flexural properties measured in the weft direction increase with respect to the ones corresponding to the flax/epoxy composite as follows: with 91.08% for the maximum value of the flexural stress σ_{max} and with 136.17% in case of Young's modulus E . The increased flexural properties in the warp direction were also recorded: with 59.97% for the maximum value of the flexural stress σ_{max} and with 182.56% in case of Young's modulus E . Table 3 shows the other flexural properties of the composite materials tested.

Theoretical results

To compute all terms of the matrix of rigidity $[\bar{Q}]$ corresponding to the flax/epoxy composite lamina, the experimental results obtained in the case of bending of the specimen whose length is parallel to either the weft or either the warp directions are used: $E'_1 = 2136.85$ MPa; $E'_2 = 1883.67$ MPa. Poisson's ratio $\nu'_{12} = 0.337$ was experimentally determined in our previous research.²³ The other Poisson's ratio $\nu'_{21} = 0.297$ was computed by using relation (6). The coefficient $\Delta' = 1 - \nu'_{12} \cdot \nu'_{21} = 0.8999 \cong 0.9$.

The required components of the rigidity matrix $[\bar{Q}]$ corresponding to the flax/epoxy lamina may be computed by using relations (5):

Table 4. Comparison of the theoretical and experimental results regarding the modulus of elasticity E_x in bending in the weft direction.

No.	Type of the composite material	Equivalent modulus of elasticity E_x in bending – weft direction (MPa)		Error (%)
		Theoretical	Experimental	
1	Flax/epoxy composite material	2136.58	2136.85	0.012
2	Glass/flax/epoxy composite material	4842.47	5046.92	4.222

$$\overline{Q}_{11} = 2374.278; \overline{Q}_{22} = 2092.967; \overline{Q}_{12} = 705.33 \quad (22)$$

By substituting relation (22) into relation (19), the theoretical modulus of elasticity corresponding to the flax/epoxy laminated beam with respect to the weft direction of the flax fabric was computed:

$$(E_x)_{flax/epoxy} = 2136.58 \text{ MPa} \quad (23)$$

To compute the modulus of elasticity E_x'' corresponding to the beam made of glass/flax/epoxy hybrid composite, it is required to compute the terms $\overline{Q}_{ij}''(i, j = 1, 2)$ of the rigidity matrix $[\overline{Q}'']$ corresponding to the glass/epoxy lamina.

For this purpose, the results obtained in the case of bending of the specimens made of glass/epoxy composite from a previous work²⁴ are used: $E_1'' = E_2'' = 6155 \text{ MPa}$. It is remembered again that the moduli of elasticity corresponding to the weft and warp directions of the glass fabric are the same due to the manufacturing particularities of such a glass fabric used (see section ‘Structure of the composite materials tested’). Poisson’s ratio $\nu_{12}'' = 0.15$ was experimentally determined in our own work.²⁵ The other Poisson’s ratio $\nu_{21}'' = 0.15$ was calculated by substituting $E_1'' = E_2''$ into relation (6). The coefficient $\Delta'' = 1 - \nu_{12}'' \cdot \nu_{21}'' = 0.9775$.

The required components of the rigidity matrix $[\overline{Q}'']$ corresponding to the glass/epoxy lamina may be computed by using relations (5):

$$\overline{Q}_{11}'' = \overline{Q}_{22}'' = 6296.6752; \overline{Q}_{12}'' = 944.5013 \quad (24)$$

Substituting relations (22) and (24) into relation (21), the theoretical modulus of elasticity corresponding to the glass/flax/epoxy laminated beam with respect to the weft direction of the flax fabric was computed:

$$(E_x)_{glass/flax/epoxy} = 4842.47 \text{ MPa} \quad (25)$$

Theoretical versus experimental results

All experimental and theoretical values regarding the equivalent modulus of elasticity E_x are summarized in Table 4.

In order to compare the results obtained by using the theoretical model of the composite beam with the ones experimentally obtained, the last column of

Table 4 shows the values of the error computed by using the formula:

$$Error = \frac{|E_x^{theory} - E_x^{exp.}|}{E_x^{theory}} \cdot 100\% \quad (26)$$

Conclusion

This article proposes a way to improve the mechanical behaviour of the composite materials reinforced with flax fabrics by reinforcing with glass fabric of two extreme layers (top and bottom layers). The improvements are significant regarding the tensile behaviour, but the increase in the mechanical properties determined in the bending test is truly spectacular. In the case of the hybrid composite, the modulus of elasticity E in bending is more than double (136.17%) and almost triple (182.56%) in the weft/warp direction, in comparison with the corresponding values recorded in the same directions of the flax/epoxy composite material. The maximum value of the flexural stress recorded in the weft direction is almost double (91.08%) in the case of the hybrid composite.

The theoretical values of the equivalent modulus of elasticity E_x are close to those determined experimentally in case of both laminated composite materials investigated since the error was equal to 0.012% in the case of the flax/epoxy composite and 4.22% in the case of glass/flax/epoxy composite. This result validates the bending beam models proposed in this work in case of both the investigated composite materials.

The relations (19) and (21) corresponding to equivalent moduli of elasticity of the bending beam models presented in this work, may be used in the case of any beam subjected to bending, made of such laminated composite materials.

Herein, the importance of replacing the reinforcement material with glass fabric in the extreme layers, is shown because the equivalent modulus of elasticity E_x increases in the case of the glass/flax/epoxy hybrid laminated composite. Moreover, the maximum stresses developed in these extreme layers during the bending and the flexural strength of the glass/epoxy layer are greater than the one corresponding to the flax/epoxy layer.

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