Theoretical and Experimental Research on the Dynamics of a 4DOF Isoglide 4-T3R1 Parallel Robot

N. Rat, M. Neagoe, and G. Gogu

Abstract The paper deals with the theoretical and experimental study of the kinematic and dynamic behaviour of a four DOF Isoglide4-T3R1 parallel robot with decoupled translation motions. The Lagrange with multipliers method was successfully applied to derive the closed-form dynamic model using the Maple software. Next, an equivalent dynamic virtual model of the Isoglide4-T3R1 parallel robot was developed and simulated in ADAMS program on different established trajectories. The theoretical dynamic models (closed-form model and ADAMS numerical approach) were validated through dynamic experimental testing carried out on the Isoglide4-T3R1 parallel robot prototype developed at French Institute of Advanced Mechanics (IFMA), France. Finally, relevant conclusions regarding the Isoglide4-T3R1 dynamic behaviour are presented.

Keywords Experimental research · Kinematic and dynamic analysis · Modelling and simulation · Parallel robot

1 Introduction

In the last years, the parallel robots used like tackle machines, manipulators, etc., began to have a great industrial importance. In comparison with serial robots, the parallel robots have some advantages like: high rigidity, good rapport payload/masse of the robot, high mechanical stiffness, etc.

The parallel robots have begun to make the object of study in priority at the last three decades. In this period, important contributions were done specially on the parallel robot modelling: kinematics [1–4] and dynamics [5].

Because the parallel robots are closed kinematics chains, constituted by a mobile platform with *n* degree of freedom, connected to the fixed base by serial or complex

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kinematics chains, the dynamic modelling proves to be complex even in the rigid body hypothesis.

Regarding the dynamic modelling of the parallel robots, different methods can be applied. A method using the Lagrange – D'Alembert formulation has been applied by Yen and Lai [11] for obtaining the dynamic equations of a 3-DOF translational parallel robot.

The virtual work principle was usefully applied by Wu et al. [12] for obtaining the dynamic equation for a 3DOF parallel robot and the obtained driving forces were optimized by applying the least-square method.

Li and Xu [13] derived the analytical dynamic model of a translational parallel robot using the dynamic equations obtained via the virtual work principle and validated on a virtual prototype with the ADAMS software.

In this paper, a kinematic and dynamic modelling for the Isoglide4-T3R1 parallel robot is presented [14]. The Lagrange method with multipliers was used to derive the closed form dynamic model in the rigid links hypothesis. Based on numerical examples, the dynamic closed form models were validated through MBS prototyping in ADAMS environments.

2 On the Kinematics of the Isoglide4-T3R1 Parallel Robot

The parallel robot Isoglide4-T3R1 (Fig. 1) has four degrees of freedom (DOF) and uncoupled translational motions. This parallel robot has four elementary legs A, B, C and D (Fig. 1), connected to the mobile platform by revolute joints and to the fixed base by linear motors (translational joint). Each elementary leg has four passive rotational joints and one active translational joint.

This parallel robot is one of a family of parallel robot with N = 2 number of overconstraints [6].

More details concerning the calculation of mobility, spatiality and number of overcontraints for this type of parallel robots are presented in [7] and [8].

The formulae for mobility calculation founded on Chebychev – Grübler-Kutzbach's criterion do not work for this parallel robot [9,10]. More general formulae, applicable to the parallel robots, have been recently proposed and demonstrated in [7] and [8].

The mobile platform has three uncoupled translations ($v_x = \dot{q}_1, v_y = \dot{q}_2, v_z = \dot{q}_3$) and one coupled rotation $\omega_y = a(\dot{q}_4 - \dot{q}_3)$ where $a = 1/r\cos(\varphi_y)$ and r = HG is the length of the mobile platform.

The variable angle φ_y (Fig. 1) depends on the difference between the position of the two z axis vertical motors (q_3 and q_4).

The moving platform velocities are related to the velocities $[\dot{q}]$ of the actuated joints by the general equation:

$$ {}^{p}\begin{bmatrix} v \\ w \end{bmatrix}_{H} = [J] \cdot \begin{bmatrix} \dot{q} \end{bmatrix} \tag{1} $$

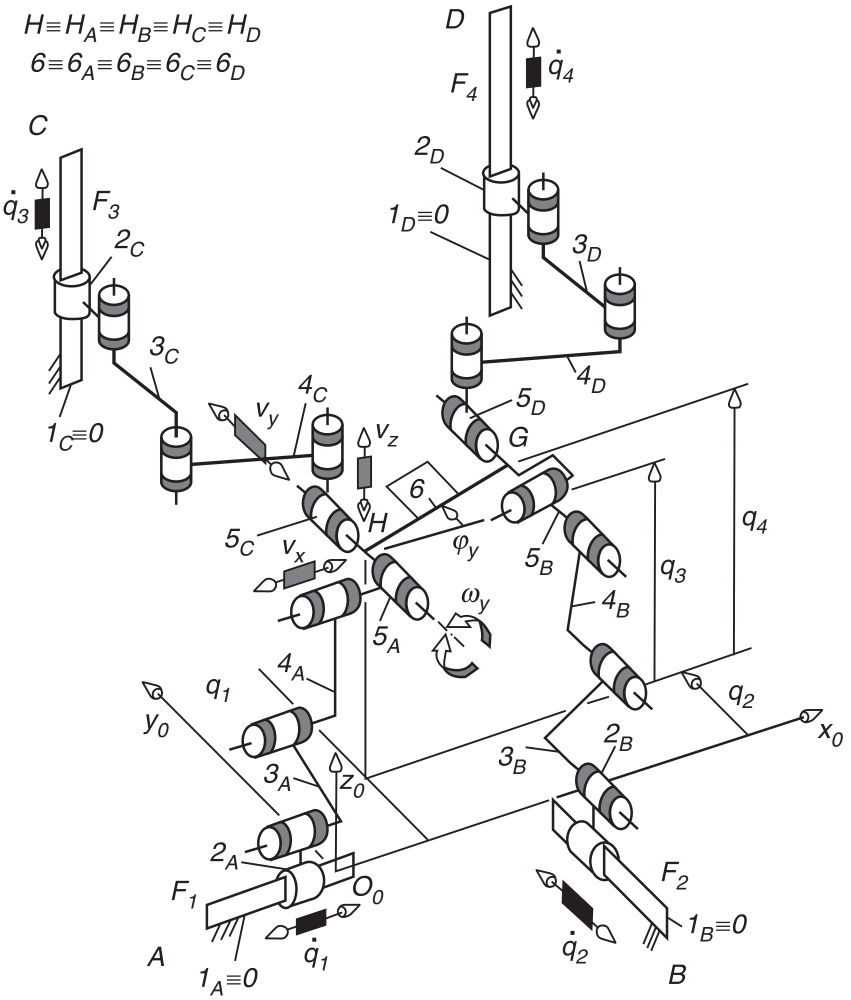

Fig. 1 The kinematical structure of Isoglide4-T3R1 parallel robot [8]

where : $[v] = \begin{bmatrix} v_x & v_y & v_z \end{bmatrix}^T$ is the velocity of the point H belonging to the moving platform; $[w] = \begin{bmatrix} w_x & w_y & w_z \end{bmatrix}^T$ is the angular velocity of the moving platform; $[J]$ is the Jacobian matrix and p is the coordinate system in which the velocities of the moving platform with respect to the fixed platform are expressed.

For the studied parallel robot the kinematical model is given by:

$$
{}_{p}\begin{bmatrix} v_x \\ v_y \\ v_z \\ w_y \end{bmatrix}_H = [J] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{q}_4 \end{bmatrix}, \tag{2}
$$

where:

$$
[J] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \frac{1}{r\cos(\varphi)} \end{bmatrix}
$$

3 Dynamic Modelling of the Isoglide4-T3R1 Parallel Robot

The dynamic modelling has been done in the premises of: (a) the gravity vector is oriented in negative sense of the z axis, and (b) no external load on the moving platform.

Considering rigid elements with distributed masses, the dynamic model was derived using the Lagrange multipliers method:

$$\sum_{i=1}^{k} \lambda_i \frac{\partial \Gamma_i}{\partial q_j} = \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} - \hat{Q}_j, \tag{3}$$

Where λ_I are the Lagrange multipliers;
q_j – displacement of the actuators;
$\hat{Q}_j$ – generalized external forces;
L – parallel robot Lagrangean:

$$L = \sum_{i=1}^{n} K_i + \sum_{i=1}^{n} P_i, \tag{4}$$

where: K_i is the kinetic energy of the link i; P_i – the potential energy of the link i.

The Lagrange multipliers λ_i are identified introducing a set of 13 geometric equations with 17 kinematic parameters (13 dependent joint variable and four independent joint variables):

- The position of the characteristic point H(X_H, Y_H, Z_H) of the mobile platform (element 6) must be the same for the arms A and C (three equations);
- The position of the point G (X_G, Y_G, Z_G) must be the same for the arms B and D (three equations);
- The distance between the point H and G is a constant r (one equation);
- The points H and G have the same coordinate Y (one equation);
- The sum of the first three rotational dependent joint variables of each of the A, C and D arms is zero (three equations);
- The sum of the first three rotational joints variables of the B arm must be equal with angle φ (one equation);
- The orientation angle φ is done by: $\sin(\varphi) = \frac{q_3 - q_4}{r}$ (one equation).

Finally, the analytical expression of the driving forces F_1, F_2, F_3 and F_4 (Fig. 1) are obtained applying the dynamic algorithm into a specific Maple application;

4 Virtual Model and Experimental Testing of the Isoglide4-T3R1 Parallel Robot

Starting from the prototype of Isoglide4-T3R1 (Fig. 2a) accomplished at LaMI laboratory of IFMA, France, a virtual ADAMS model was obtained (Fig. 2b).

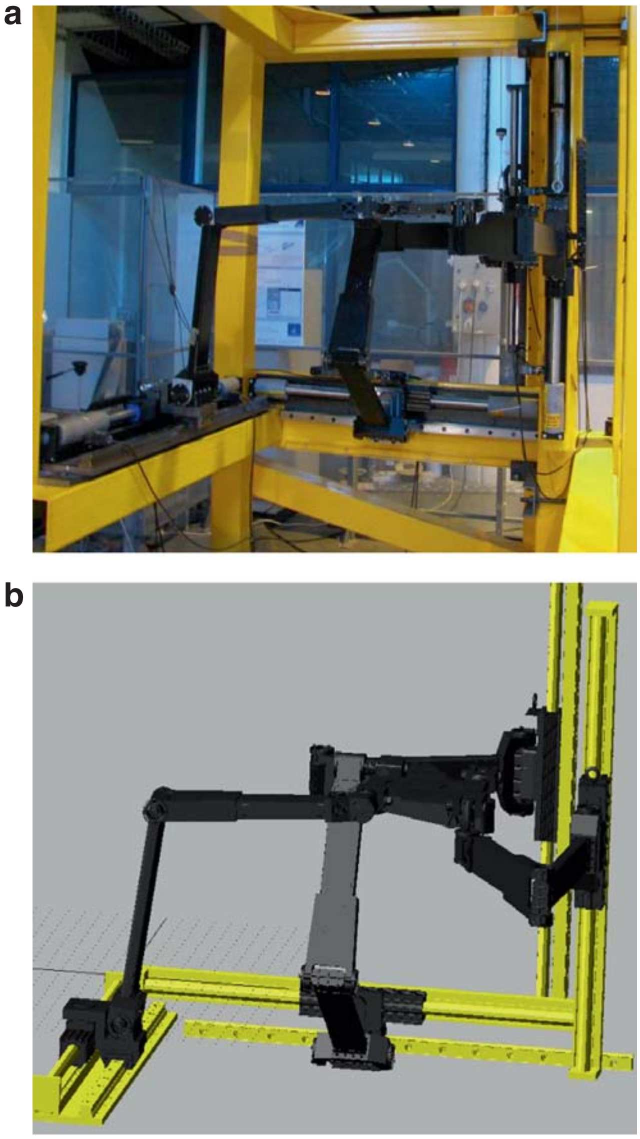

Fig. 2 The prototype (**a**) and the virtual model (**b**) of the Isoglide4 – T3R1 parallel robot

4.1 Isoglide4-T3R1 CAD Model

The real prototype was made using the parts designed in CATIA software.

The virtual prototype (Fig. 3) was developed in ADAMS software using the parts imported from CATIA. For each part, the same inertial proprieties were taken in consideration (masses and inertial moments). In this way, the same forms, the same materials and the same proprieties for each part from the CAD model and real prototype are maintained.

The CAD model has the same reference coordinate system as the physical prototype, with the origin O_0 placed at intersection of the axes of the motors M_1 and M_2 (Fig. 3).

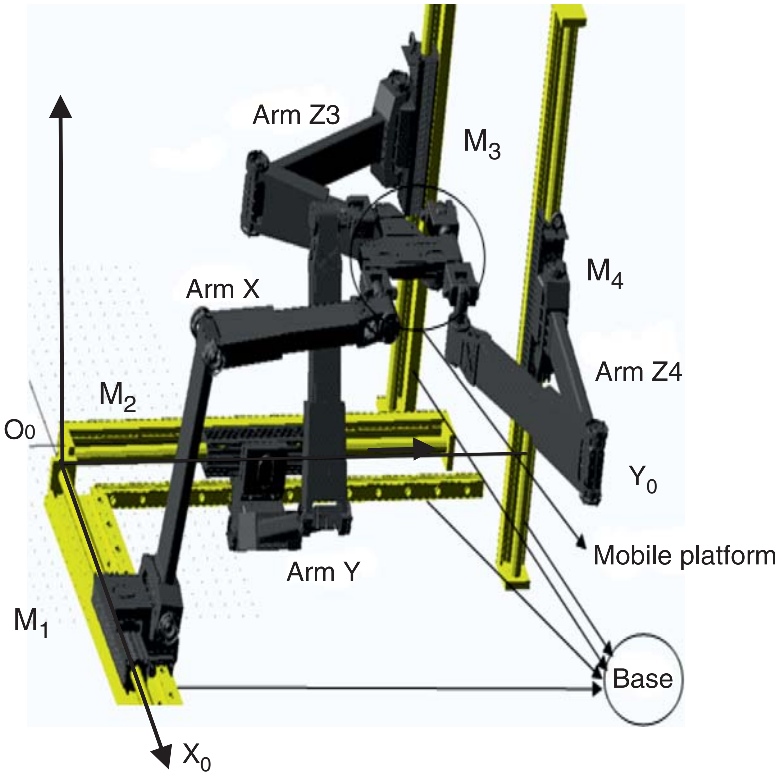

Fig. 3 ADAMS model of the Isoglide4 – T3R1 parallel robot: general view and parameterization

4.2 *Dynamic Experimental Testing of the Isoglide4-T3R1 Parallel Robot*

An experimental testing was made with physical prototype (Fig. 2a). In parallel, a CAD model (in ADAMS) was developed for comparing the obtained results.

For testing the robot, a linear trajectory (with fifths degree polynomial law of time) was taken into consideration. The choice displacement is of 0.2 m for motor M_1 with acceleration maximum of 1 m/s^2 during 1.062 s.

The experimental displacement of the motor M_1, presented in Fig. 4, follows closely the commanded displacement. The perturbation effects of the system become perceptible for the speed and acceleration (obtained by numerical derivation), presented in Figs. 5 and 6.

For comparison with the ADAMS model, the next steps are taken in consideration:

1. Filtration of the obtained acceleration from experimental tests – Fig. 6, using MATLAB software;
2. Integration of the filtered acceleration (Fig. 7) to obtain filtered speed and displacement (Fig. 8);
3. The filtered displacements are inserted in ADAMS model as movement law of time for the motors;

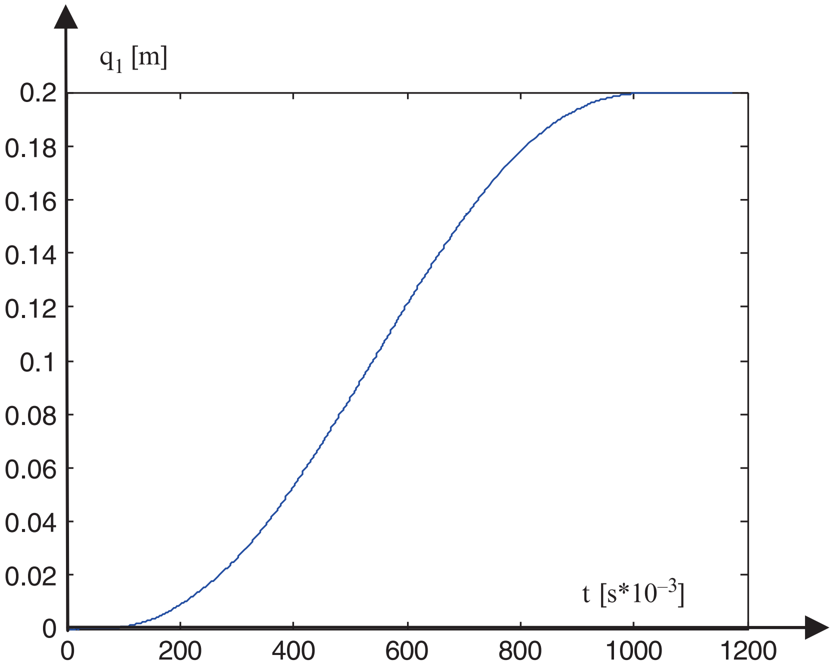

Fig. 4 The experimental obtained displacement for the linear motor M_1

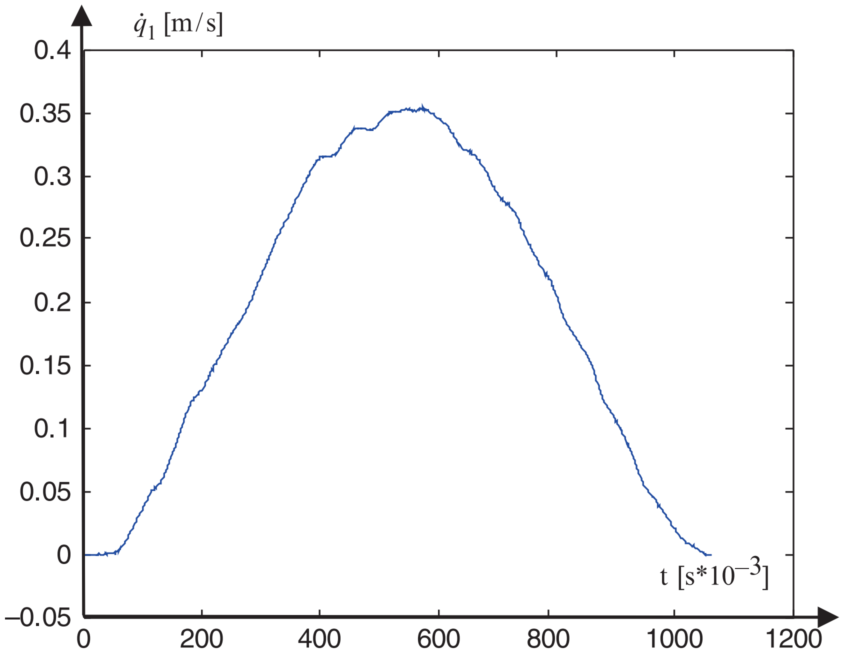

Fig. 5 The experimental obtained speed for the linear motor M_1

4. Establishing of the experimental force $F_1 = ct*U(t)$, where U(t) is the measured tension;
5. The filtration of the obtained driving force F_1 (Fig. 9);
6. Comparing the obtained results from experimental tests and ADAMS model (Fig. 10).

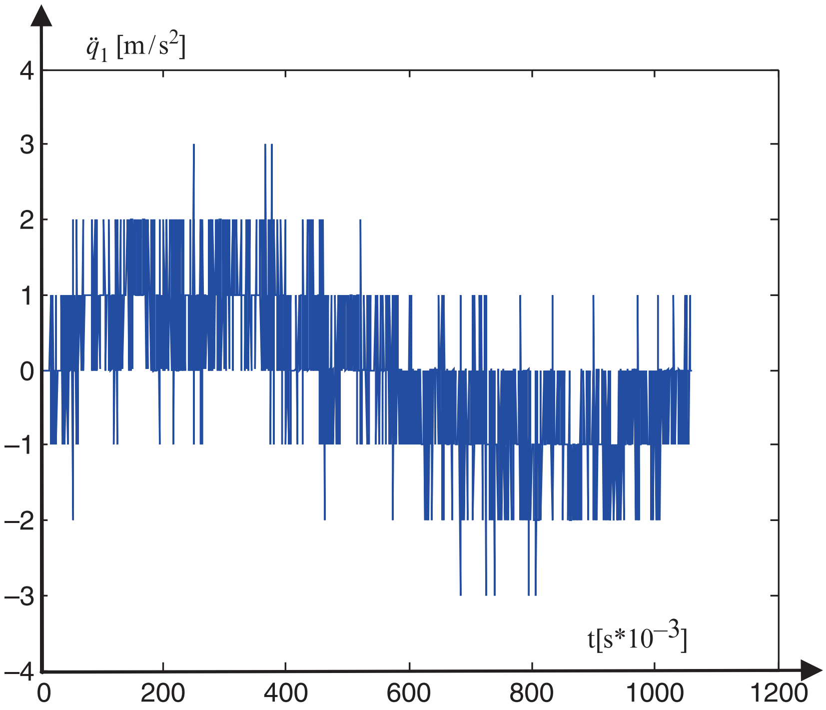

Fig. 6 The experimental obtained acceleration for the linear motor M_1

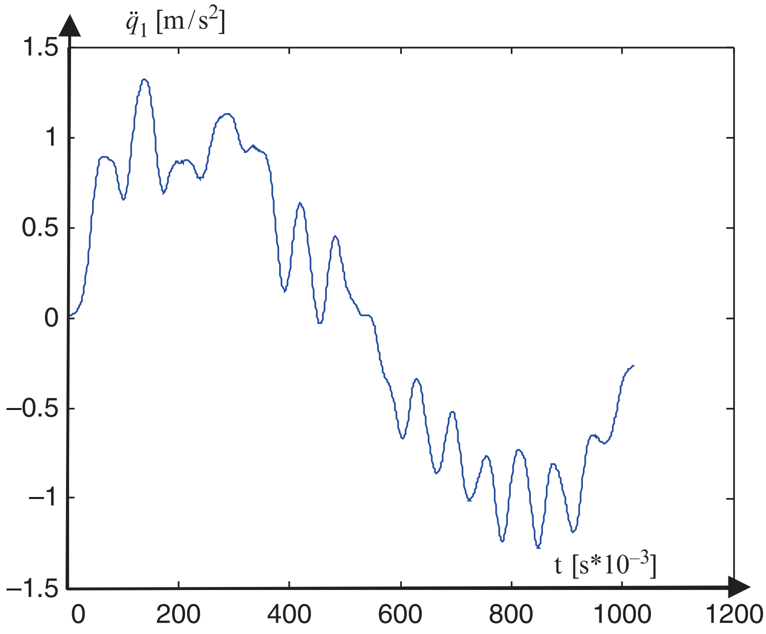

Fig. 7 The filtrated acceleration of the linear motor M_1

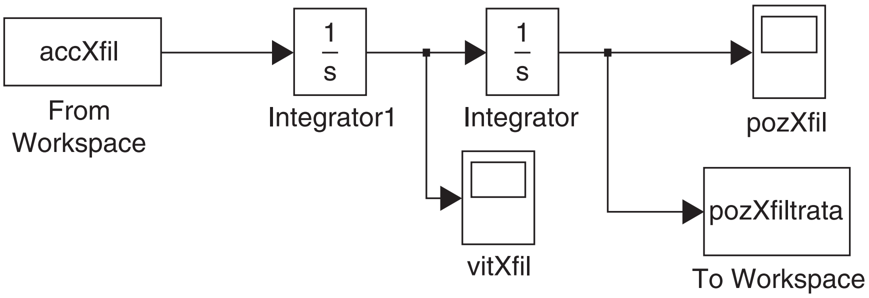

Fig. 8 The integration of the filtered acceleration

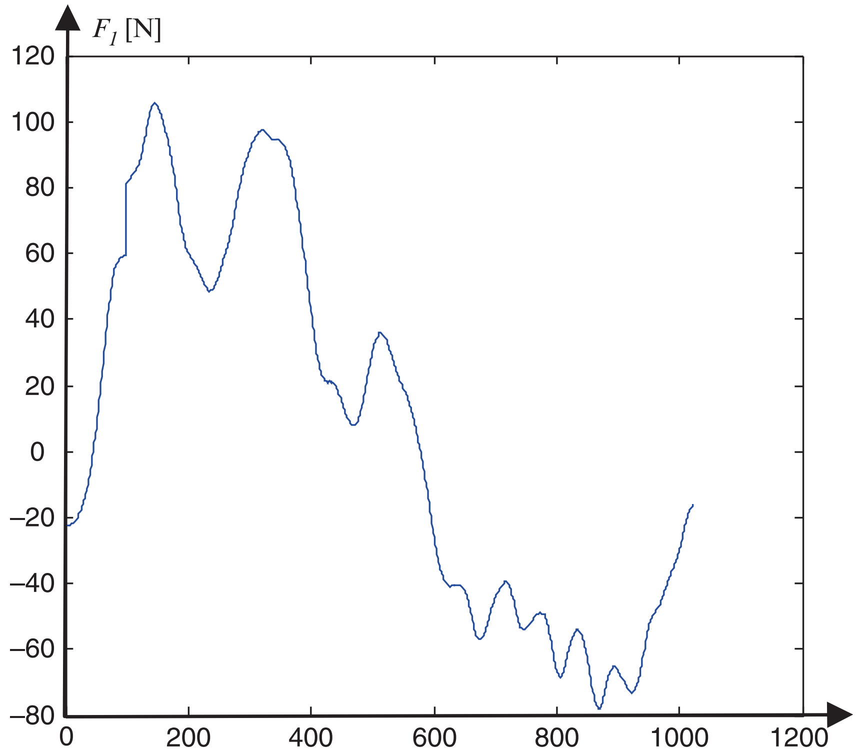

Fig. 9 The experimental filtered driving force F_1

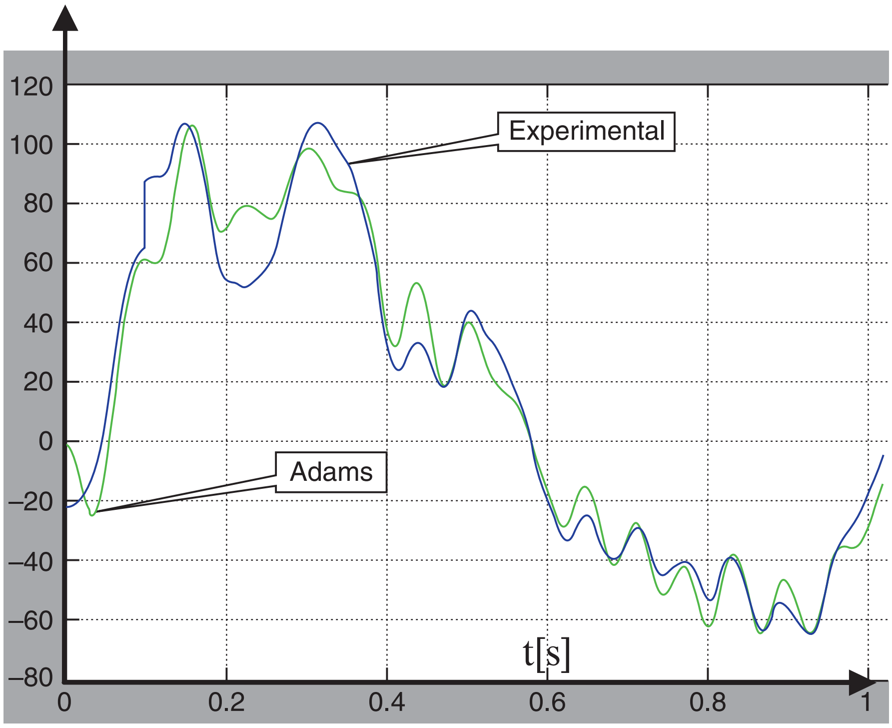

Fig. 10 The driver force F_1 obtained experimentally (see also Fig. 9) and using ADAMS simulation

5 Conclusions

The study highlights the following conclusions:

- An analytical dynamic model was obtained using Maple software;
- Even the analytical dynamic model is complex, the method used can be usefully applied to derive the dynamical model of any parallel robots;
- A virtual CAD model was done in ADAMS environment, obtaining a CAD model in concordance with the physical model;
- Dynamic experimental testing was done on a physical prototype from IFMA, France, obtaining a good concordance with the results of the theoretical close-form and virtual CAD models.

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